Comparing the Derivation of Modal Domains and Strengthened Meanings

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Abstract. The derivation of strengthened meanings as proposed by Bar-Lev and Fox (2017, 2020) and the derivation of modal domains as proposed by Kratzer (1977, 1981, 1991) both involve an "inclusion" step of assigning *true* to as many propositions in a given set as possible. In the case of strengthened meanings, this set contains the scalar alternatives. In the case of modal domains, it contains the propositions in the ordering source. In this note, we explicate what is common and what is distinct between the two inclusion procedures. We then point out that the formal distinction makes no empirical difference for the cases of strengthened meaning so far considered in the literature. We conjecture that this fact holds generally for all cases of strengthened meaning.

Keywords: Modality · Exhaustification · Innocent inclusion · Cell identification · Alternatives

1 Two Steps of Exhaustification

1.1 Exclusion

The "grammatical approach to implicatures" takes the strengthened meaning of a sentence p, i.e. the conjunction of p and its implicatures, to result from applying an exhaustivity operator exh to p (cf. Fox 2007; Chierchia et al. 2012). Fox (2007) proposes that $exh(p)$ assigns *true* to p, the "prejacent", and assigns *false* to each of the "innocently excludable alternatives", henceforth "IE alternatives", of p.[1] We present Fox's 2007 proposal in (1), where A_p^{IE} is the set of IE alternatives of p and $\bigvee S$ is the proposition that at least one member of S is true, for any

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[1] More precisely, $exh(p)$ assigns *true* to p and assigns *false* to each of the IE alternatives of p which are *relevant*. For the purpose of this discussion, we will make the simplifying assumption that the alternatives are all relevant. We do not believe this assumption affects our argument..

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set S of propositions.[2] Innocent exclusion is defined in (2), where A_p is the set of alternatives of p.

(1) Fox's (2007) proposal

a. The strengthened meaning of p is expressed by $exh(p)$

b. $exh(p) \Leftrightarrow_{\text{def}} p \wedge \left(A_p^{IE} \neq \emptyset \rightarrow \neg \bigvee A_p^{IE}\right)$

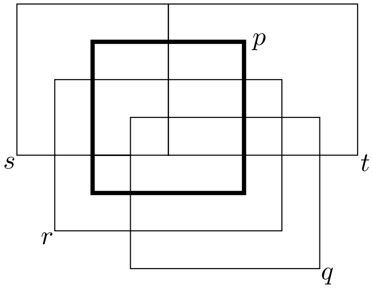
(2) Fox's (2007) definition of A_p^{IE}

(i) Take all maximal sets of propositions from A_p which can be assigned *false* consistently with p

(ii) $q \in A_p^{IE}$ iff q is in all such sets

To illustrate, consider the Venn diagram below. Let p be the prejacent and q, r, s, t and p itself be its alternatives.[3] Logical relations are represented spatially in the familiar way. Thus, we have $p \Rightarrow (s \vee r \vee t)$, $(s \wedge t) \Rightarrow \bot$, for example.

(3)



The maximal sets of propositions from A_p which can be assigned *false* consistently with p are listed in (4).

(4) a. $\{q, r, s\}$
b. $\{q, r, t\}$
c. $\{q, s, t\}$

Note that neither $\{r, s, t\}$ nor $\{q, r, s, t\}$ is listed, as $(\neg r \wedge \neg s \wedge \neg t) \Rightarrow \neg p$. Now, looking at (4), we see that only q is a member of all three sets. Thus, only q is an IE alternative of p, which means $exh(p) \Leftrightarrow p \wedge \neg q$. This proposition is indicated by the gray area in (5).

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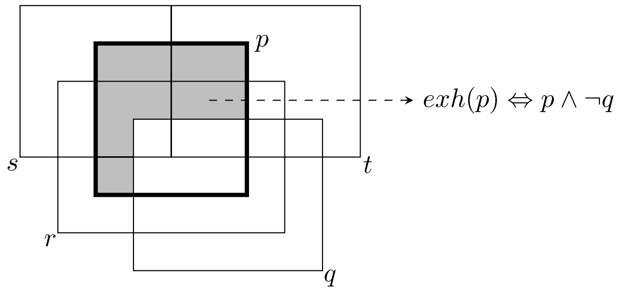
[2] The attentive reader will notice that the definition in (1b) contains a redundancy. Specifically, $A_p^{IE} \neq \emptyset \rightarrow \neg \bigvee A_p^{IE}$ is equivalent to $\neg \bigvee A_p^{IE}$, as $\bigvee \emptyset$ is the contradiction. The intuition which we want this redundant formulation to reflect is that the computation proceeds only under the condition that the relevant set of altermatives is not empty. That condition is logically idle for this case but not for all of the cases which we will discuss..

[3] We assume, as is standard, that every sentence is an alternative of itself (cf. Fox and Katzir 2011).

(5)



The process of exhaustification, as represented by *exh*, can therefore be described, informally, as that of trying to assign *false* to as many alternatives as possible, preserving consistency with the prejacent.

1.2 Inclusion

Bar-Lev and Fox (2017, 2020), henceforth BLF, propose that the exhaustivity operator be modified. Specifically, they argue that it should be not *exh* but exh', as defined in (6b), where *exh* remains as defined in (1b), A_p^{II} is the set of "innocently *in*cludable alternatives", henceforth "II alternative", of p, and $\bigwedge S$ is the proposition that every member of S is true, for any set S of propositions.[4] Innocent inclusion is defined in (7).

(6) BLF's proposal

a. The strengthened meaning of p is expressed by $exh'(p)$

b. $exh'(p) \Leftrightarrow_{\text{def}} exh(p) \wedge \left(A_p^{II} \neq \emptyset \rightarrow \bigwedge A_p^{II}\right)$

(7) BLF's definition of A_p^{II}

(i) Take all maximal sets of propositions from A_p which can be assigned *true* consistently with $exh(p)$

(ii) $q \in A_p^{II}$ iff q is in all such sets

What $exh'(p)$ does, then, is assign *true* to $exh(p)$ and also assign *true* to each of the innocently includable alternatives of p.[5]. Consider, again, the Venn diagram in (3). Let us ask which among q, r, s, t and p itself is an II alternative of p. The maximal sets of propositions from A_p which can be assigned *true* consistently with $exh(p)$ are listed in (8).

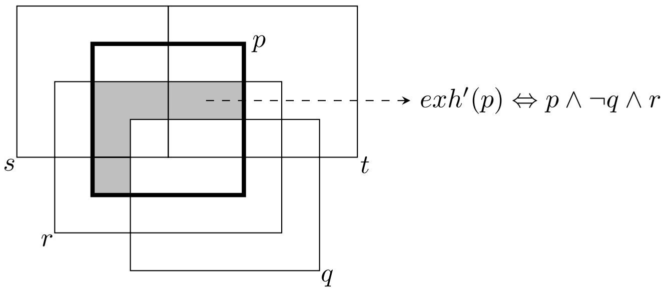
(8) a. $\{p, r, s\}$
b. $\{p, r, t\}$

[4] Again, there is redundancy in (6b), as $\bigwedge \emptyset$ is the tautology. See note 2.

[5] Note that BLF claims that II alternatives are assigned *true* obligatorily (cf. Bar-Lev and Fox 2017, 111). Thus, the inferences associated with them cannot be cancelled by them being considered irrelevant, as is possible in the case of IE alternatives (see note 1)..

Note that neither $\{p, s, t\}$ nor $\{p, r, s, t\}$ is listed: since $s \wedge t$ is contradictory, no set containing s and t is consistent. Now, looking at the two sets in (8), we see that only p and r are members of both. This means only p and r are II alternatives of p, and that $exh'(p) \Leftrightarrow exh(p) \wedge p \wedge r \Leftrightarrow p \wedge \neg q \wedge r$. This proposition is indicated by the gray area in (9).

(9)



The process of exhaustification, as represented by exh', can therefore be described, informally, as that of (i) trying to assign *false* to as many alternatives as possible, preserving consistency with the prejacent, and then (ii) trying to assign *true* to as many alternatives as possible, preserving consistency with the output of (i). Thus, $exh'(p)$ is a strengthening of $exh(p)$. We can see this by comparing the gray area of (5) with the gray area of (9): the latter is a subpart of the former.

1.3 Empirical Motivation for *exh′*

BLF present a series of empirical arguments for identifying the strengthened meaning of p with $exh'(p)$ instead of $exh(p)$. Given the scope of this note, we will recite only one. The reader is invited to consult Bar-Lev and Fox (2017, 2020) to learn about the others.

The relevant data point is the sentence in (10), which has been argued to license the inferences in (10a) and (10b) (cf. Chemla 2009).

(10) No student is required to solve both problem A and problem B $\neg\exists x \Box(Px \wedge Qx)$

a. $\rightsquigarrow$ No student is required to solve problem A $\neg\exists x Px$

b. $\rightsquigarrow$ No student is required to solve problem B $\neg\exists x Qx$

The syntactic analysis of (10) at the relevant level, i.e. its Logical Form, is assumed to be something like (11).

(11) $[_\alpha$ no student λ_x $[_\beta$ is required to $[_\gamma$ t_x solve A and t_x solve B$]]]$

Given that **required** and **and** are both strong scalar items, exhaustifying β or γ will be semantically inconsequential.[6] The only scope site left for possibly non-vacuous exhaustification is the matrix node, which means the exhaustivity

[6] Because $exh(p \wedge q) = exh'(p \wedge q) = p \wedge q$, and $exh(\Box p) = exh'(\Box p) = \Box p$.

operator must be applied to α. BLF take the alternatives of α to be derived from α by replacing **no** ($\neg\exists$) with **not every** ($\neg\forall$), **and** with **or**, γ with its individual conjuncts, and α with itself.[7] We then have (12).

(12) a. Prejacent: $\neg\exists x\Box(Px \wedge Qx)$
b. Alternatives: $\neg\exists x\Box(Px\wedge Qx)$, $\neg\exists x\Box(Px\vee Qx)$, $\neg\exists x\Box Px$, $\neg\exists x\Box Qx$, $\neg\forall x\Box(Px \wedge Qx)$, $\neg\forall x\Box(Px \vee Qx)$, $\neg\forall x\Box Px$, $\neg\forall x\Box Qx$
c. IE alternatives: $\neg\exists x\Box(Px \vee Qx)$, $\neg\forall x\Box(Px \vee Qx)$
d. II alternatives: $\neg\exists x\Box(Px \wedge Qx)$, $\neg\exists x\Box Px$, $\neg\exists x\Box Qx$, $\neg\forall x\Box(Px \wedge Qx)$, $\neg\forall x\Box Px$, $\neg\forall x\Box Qx$

The results of applying exh and exh' to α, with the redundancies removed, amount to (13a) and (13b), respectively.

(13) a. $exh\big(\neg\exists x\Box(Px \wedge Qx)\big) \Leftrightarrow \neg\exists x\Box(Px \wedge Qx) \wedge \forall x\Box(Px \vee Qx)$
b. $exh'\big(\neg\exists x\Box(Px \wedge Qx)\big) \Leftrightarrow \neg\exists x\Box(Px \wedge Qx) \wedge \forall x\Box(Px \vee Qx) \wedge \neg\exists x\Box Px \wedge \neg\exists x\Box Qx$

We can see that the attested inferences can be derived with exh' but not with exh. More specifically, there is no way to derive these inferences with exh, but there is one way to derive them with exh'.[8]

2 A More Inclusive Inclusion

2.1 Conceptual Motivation for Inclusion

BLF mention a "possible underlying conception" which they say has "guided [their] thinking".

(14) Possible underlying conception (Bar-Lev and Fox 2020, 186)
Exhaustifying p with respect to a set of alternatives C should get us as close as possible to a cell in the partition induced by C

We quote from (Bar-Lev and Fox 2020, 186): "[...] [T]he goal of [the exhaustivity operator] is to come as close as possible to an assignment of a truth value to every alternative, i.e., to a cell in the partition that the set of alternatives induces

[7] The assumption that **no** ($\neg\exists$) alternates with **not every** ($\neg\forall$) is based on the analysis of **no** which decomposes it into **not** and **some** (cf. Zeijlstra 2004; Penka 2011), and on the view about alternative generation according to which negation is not replaced (cf. Romoli 2012). See Bar-Lev and Fox (2020, 198, note 32) on this point. Also, see Bar-Lev and Fox (2020, 198–200) for some independent reasons to assume that $\Box$ does not alternate with $\Diamond$ in this case.

[8] BLF point out that recursive application of exh does not help (cf. Bar-Lev and Fox 2020, 196). Note, also, that there is, in addition to the inferences in (10a) and (10b), another inference derived in (13b), namely $\forall x\Box(Px \vee Qx)$. This inference is really optional, since it should only arise if the alternative $\neg\forall x\Box(Px \vee Qx)$ is considered relevant, which it does not have to be. Again, we consider, for the purpose of this note, all alternatives to be relevant. (See note 1 and 5.).

[...] [The exhaustivity operator] is designed such that, when possible, it would yield a complete answer to the question formed by the set of alternatives. If this conception is correct, one would think that [it] shouldn't only exclude, i.e., assign *false* to as many alternatives as possible, but should also include, i.e., assign *true* to as many alternatives as possible once the exclusion is complete".

Looking at (5) and (9), we can see clearly how this idea plays out. The proposition expressed by $exh(p)$ consists of five cells in the partition induced by the alternatives, while the proposition expressed by $exh'(p)$ consists of three of these five cells. Thus, exhautification by exh' gets us closer to a single cell than exhaustification by exh.

2.2 Introducing *exh''*

Nothing in the definitions of innocent exclusion and innocent inclusion rules out the possibility of alternatives which are neither innocently excludable nor innocently includable. We will call such alternatives the "remaining alternatives", or "R alternatives" for short. Now let us entertain the hypothesis in (15), where A_p^R is the set of R alternatives of p and exh' is as defined in (6b).[9]

(15) Hypothesis

a. The strengthened meaning of p is expressed by $exh''(p)$

b. $exh''(p) \Leftrightarrow_{\text{def}} exh'(p) \wedge \left(A_p^R \neq \emptyset \rightarrow \bigvee A_p^R\right)$

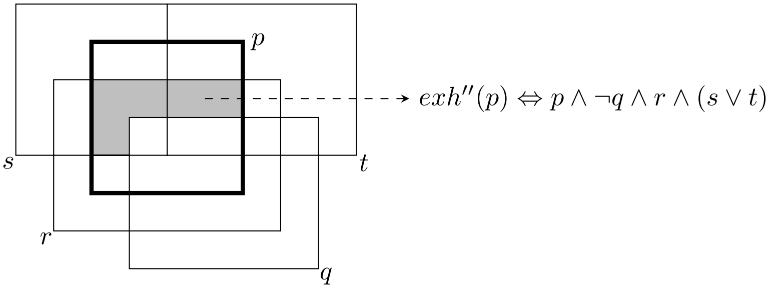
(16) Definition of A_p^R

$q \in A_p^R$ iff $q \in A_p \wedge q \notin A_p^{IE} \wedge q \notin A_p^{II}$

The new exhaustivity operator we are considering, exh'', involves a more "inclusive" inclusion than exh'. Specifically, $exh''(p)$ not only includes the II alternatives by assigning *true* to each of them, but also "includes" the R alternatives by assigning *true* to their disjunction. Thus, $exh''(p)$ is a strengthening of $exh'(p)$, which means exh'' actually comes closer to BLF's "underlying conception" of exhaustification than exh'. We can see this by looking at (17), where the gray area represents $exh''(p)$.

[9] Note that the condition that A_p^R not be empty is significant here. We want to capture the intuition that if there is no R alternative, the system would just output $exh'(p)$. Specifically, we do want it to not output the contradiction in case A_p^R is empty, which is what would happen if $exh''(p)$ were defined as $exh'(p) \wedge \bigvee A_p^R$.

(17)



Comparing (17) to (9), we see that the proposition expressed by $exh'(p)$ consists of three cells and the proposition expressed by $exh''(p)$ consists of two of those three cells. Thus, $exh''(p)$ is closer to a complete answer of the question formed by the set of alternatives than $exh'(p)$.

2.3 A Resemblance

Kratzer (1977, 1981, 1991) propose that modality is "double relative". Specifically, the quantification domain D of a modal operator is specified in terms of two sets of propositions, a "modal base" M and an "ordering source" O,[10] in the following way.

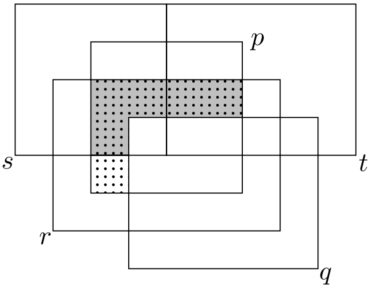
(18) Derivation of D from M and O
  a. Take all maximal sets of propositions from O which can be assigned *true* consistently with $\bigwedge M$
  b. D is the result of conjoining $\bigwedge M$ with
    (i) propositions that are in all such sets
    (ii) the disjunction of the remaining propositions in O

A necessity statement $\Box a$ is then true iff a is entailed by D, and a possibility statement $\Diamond a$ is true iff a is consistent with D.

As we can see, the two steps (18b-i) and (18b-ii) resemble the inclusion of II and R alternatives, respectively. Thus, if we identify $\bigwedge M$ with $exh(p)$ and O with $A_p - A_p^{IE}$, then we can identify D with $exh''(p)$. Let us, again, use our Venn diagram to illustrate. Suppose $M = \{p, \neg q\}$ and $O = \{r, s, t\}$. Then D is the gray area, which corresponds to $exh''(p)$. Importantly, D is not the dotted area, which corresponds to $exh'(p)$.

[10] Technically, the two sets of propositions are values of the modal base and the ordering source at the evaluation world, as these are functions from worlds to sets of propositions. The reader is invited to consult Kratzer (1977, 1981, 1991) for a more precise and sophisticated presentation of her theory. Relevant secondary literature includes von Fintel and Heim (2011); Frank (1996), among others.

(19)



$M = \{p, \neg q\}$
$O = \{r, s, t\}$
$D = p \wedge \neg q \wedge r \wedge (s \vee t)$
$D \neq p \wedge \neg q \wedge r$

We will illustrate with an example. Let us give the following meanings to r, s, and t.[11]

(20) a. r = John volunteered as poll watcher
b. s = John voted Republican
c. t = John voted Democrat

And let it be common ground that $p \wedge \neg q$.[12] This will be the modal base. Suppose that John's father says he voted Republican (s), John's mother says he voted Democrat (t), and both of John's parents say he volunteered as poll watcher (r). This will be the ordering source. Now consider the following sentences.

(21) a. In view of what his parents say, it is possible that John volunteered as poll watcher and voted Republican $\Diamond(r \wedge s)$
b. In view of what his parents say, it is possible that John volunteered as poll watcher and did not vote $\Diamond(r \wedge \neg s \wedge \neg t)$

If $D = p \wedge \neg q \wedge r$, the dotted area, we expect both (21a) and (21b) to be true, as both $r \wedge s$ and $r \wedge \neg s \wedge \neg t$ are consistent with $p \wedge \neg q \wedge r$. If $D = p \wedge \neg q \wedge r \wedge (s \vee t)$, the gray area, we expect (21a) to be true and (21b) to be false, as $r \wedge s$ is consistent with $p \wedge \neg q \wedge r \wedge (s \vee t)$ but $r \wedge \neg s \wedge \neg t$ is not. Our intuition is that (21a) is true and (21b) is false. This fact constitutes evidence that D is $p \wedge \neg q \wedge r \wedge (s \vee t)$, the gray area, and not $p \wedge \neg q \wedge r$, the dotted area.

3 A Conjecture

The Kratzerian inclusion of the ordering source involves including the disjunction of propositions which are not "innocently includable". Thus, it resembles the inclusion step of exh'', not that of exh'. Our judgement about (21a) and (21b) confirms that Kratzer is correct.

[11] We will assume that one can vote for only one party, and that the only choices are Republican and Democrat.

[12] In other words, let what we know be consistent with r, s, t, $r \wedge s$, $r \wedge t$, and let it assymetrically entail $r \vee s \vee t$. For concreteness, we can take this body of information to be the proposition that John lived in D.C and either voted or volunteered as poll watcher.

What about exh' and exh'' themselves? We have seen how they differ formally. Are there cases which distinguish them empirically? Let us look again at the definition of exh''.

(22) $$exh''(p) \Leftrightarrow_{\text{def}} exh'(p) \wedge \left(A_p^R \neq \emptyset \rightarrow \bigvee A_p^R\right)$$

Logically, $exh'(p)$ and $exh''(p)$ will be equivalent in two scenarios.

(23) $exh'(p) \Leftrightarrow exh''(p)$ iff either (a) or (b) holds
  a. $exh'(p) \Rightarrow \bigvee A_p^R$
  b. $A_p^R = \emptyset$

Consider (23a) first. This scenario is instantiated by plain disjunctions such as (24).

(24) John talked to Mary or Sue ($p \vee q$)
  a. Alternatives: $p \vee q, p, q, p \wedge q$
  b. IE alternatives: $p \wedge q$
  c. II alternatives: $p \vee q$
  d. R alternatives: p, q
  e. $exh'(p \vee q) \Leftrightarrow (p \vee q) \wedge \neg(p \wedge q)$

Now consider (23b). This scenario is exemplified by (10), discussed in Sect. 1.3. As the reader can see from (12), the IE and II alternatives of (10) exhaust the set of its alternatives. Thus, there are no R alternatives left.

Another case where every alternative is either IE or II is one involving the scalar items **all**, **many**, **some**. Consider the three sentences in (25).

(25) a. John did all of the homeworks
  b. John did many of the homeworks
  c. John did some of the homeworks

Each of these sentences has all three as alternatives. This means that for (25a), every alternative is II. For (25b), (25a) is IE while (25b) and (25c) are II. And for (25c), (25a) and (25b) are IE while (25c) is II.

When will $exh'(p)$ and $exh''(p)$ not be equivalent? Obviously when both (23a) and (23b) are false. Has a case been discussed in the literature which exemplifies this possibility? The answer to this question, we believe, is negative. As far as we know, all cases considered in the literature on exhaustification so far, including those discussed in Bar-Lev and Fox (2017, 2020), are either an instance of (23a) or an instance of (23b). We conjecture that this holds generally for all cases of exhaustification.

(26) Strengthened Meaning Conjecture (SMC)
  There is no sentence p in natural language such that the strengthened meaning of p is $exh'(p)$ but not $exh''(p)$

From our discussion in Sects. 2.1 and 2.3, it is clear that the exhaustivity operator could in principle be exh'', not exh', and that the distinction between exh'' and exh' could in principle make an empirical difference. We could imagine the facts about semantic strengthening to be such that they adjudicate between the two different inclusion procedures involved in exhaustification, just as facts about modality do with respect to the ordering source. So what is missing? We believe that SMC will follow given a complete theory of alternatives. In other words, we believe that such a theory would rule out the scenario in (3) as a grammatical impossibility. We therefore formulate the following challenge for future research.

(27) Challenge
Construct the theory of alternatives so that SMC follows

4 Conclusion

Bar-Lev and Fox (2017, 2020) propose to add inclusion to exhaustification. In addition to providing empirical arguments for their proposal, they also note that the addition makes conceptual sense given the natural understanding of semantic strengthening as an attempt by the grammar to get as close as possible to a complete answer to the question formed by the set of alternatives. We discuss a variant of inclusion which would better represent this attempt than the variant proposed by Bar-Lev and Fox. We show that the new variant resembles the inclusion of ordering sources in Kratzer's (1977, 1981, 1991) theory of modality. We point out that the two variants end up being empirically equivalent for the cases of strengthened meaning so far considered in the literature. We conjecture that this equivalence is a general fact about exhaustification, and pose the challenge of deriving it for future research.

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