Comparing the derivation of modal domains and strengthened meanings

Overall summary – The derivation of modal domains in the theory proposed by Kratzer (1981, 1991) and the derivation of exhaustified meanings in the theory proposed by Bar-Lev and Fox (2020) both involve conjoining a proposition with as many propositions in a set as possible. In the case of modal domains, the set in question is the ordering source. In the case of exhaustified meanings, it is the set of scalar alternatives. In this talk, we provide a reformulation of the two theories which brings out clearly what distinguishes Kratzer's procedure from Bar-Lev and Fox's. We argue that the distinction is one without difference: the empirical cases which Bar-Lev and Fox presented to support their theory turn out to be those in which the two procedures yield the same results. We note that Kratzer's procedure actually comes closer to the process of "cell identification" which Bar-Lev and Fox considered to be a motivation for their procedure. We also point out that the equivalence of the two procedures is not a consequence of current theories of alternatives, and hence, that such theories may be missing a generalization.

1. Reformulation of Kratzer's theory – Kratzer (1981, 1991) derives the domain of a modal operator in terms of two sets of propositions: B, the modal base at the evaluation world, and O, the ordering source at the evaluation world. Let h be the function which maps a proposition p and a set of propositions C to a set of subsets of C each of which contains as many members of C as can be consistently conjoined with p.

(1) $h(p, C) := \{C' \mid C' \text{ is a maximal subset of } C \text{ such that } p \wedge \bigwedge C' \text{ is consistent}\}$

And let $b := \bigwedge B$, the conjunction of all propositions in B. Then,

(2) $d := b \wedge \bigwedge \cap h(b, O) \wedge \bigvee(\cup h(b, O) - \cap h(b, O))$

will represent the modal domain. Thus, $\Box p \Leftrightarrow \cap d \subseteq p$ and $\Diamond p \Leftrightarrow \cap d \not\subseteq \neg p$.

2. Reformulation of Bar-Lev & Fox's theory – Bar-Lev and Fox (2020) derives the meaning of $exh(A)(p)$, which expresses the "strengthened" meaning of p given the set of alternatives A, in terms of $IE(p, A)$, the set of "innocently excludable" alternatives of p in A, and $II(p, A)$, the set of "innocently includable" alternatives of p in A.

(3) $exh(A)(p) \Leftrightarrow \bigwedge\{r \mid r \in II(p, A)\} \wedge \bigwedge\{\neg q \mid q \in IE(p, A)\}$

Let e be the conjunction of p and the negation of the IE alternatives, i.e. $e := p \wedge \bigwedge\{\neg q \mid q \in IE(p, A)\}$. Then,

(4) $m := e \wedge \bigwedge \cap h(e, A)$

will be the meaning of $exh(A)(p)$, where the function h is as defined in (1).

3. Comparison – As we can see from (2) and (4), there are similarity and difference between how d relates to b and O on the one hand and how m relates to e and A on the other. The similarity is this: d entails the conjunction of propositions contained in all members of $h(b, O)$, and m, in the same way, entails the conjunction of propositions contained in all members of $h(e, A)$. The difference is this: d entails the disjunction of propositions contained in some but not all members of $h(b, O)$, but m, on the contrary, does **not** entail the disjunction of propositions contained in some but not all members of $h(e, A)$.

4. A distinction without a difference – Let us now imagine a meaning m' for $exh(A)(p)$ which makes the step from e to $exh(A, p)$ completely parallel to the step from b to d in Kratzer's theory.

(5) $m' =_{def} e \wedge \bigwedge \cap h(e, A) \wedge \bigvee(\cup h(e, A) - \cap h(e, A))$

Then, let us ask the question: What tells us that $exh(A)(p)$ is m instead of m'? We believe the answer is "nothing". Bar-Lev and Fox (2020) discussed several cases which are explained by the assumption that $exh(A)(p) = m$. It turns out, however, that these cases will also be explained by the assumption that $exh(A)(p) = m'$, because they are instances in which m and m' are equivalent. Specifically, they are cases where (6) holds.

(6) $e \wedge \bigwedge \cap h(e, A) \Rightarrow \bigvee \big(\cup h(e, A) - \cap h(e, A)\big)$

4.1. Plain disjunctions – Consider $exh(A)(p)$ where $p =$ *John talked to Mary or Sue*. In this case, we have $\cup h(e, A) - \cap h(e, A) = \{mary, sue\}$, hence $\bigvee\big(\cup h(e, A) - \cap h(e, A)\big) = \bigvee\{mary, sue\} = mary \vee sue$. Since $e = (mary \vee sue) \wedge \neg(mary \wedge sue)$, (6) holds, which means $m = m'$.

4.2. Free choice disjunctions – Consider $exh(A)(p)$ where $p =$ *John is allowed to talk to Mary or Sue*. In this case, we have $\cup h(e, A) - \cap h(e, A) = \emptyset$, which means $\bigvee\big(\cup h(e, A) - \cap h(e, A)\big) = \bigvee\emptyset = \top$, as the disjunction of every proposition in $\emptyset$ is the disjunction of every proposition, i.e. $\top$. Since $\top$ is entailed by every proposition, this case is also one where (6) holds, i.e. where $m = m'$.

4.3. Other cases – In Bar-Lev & Fox's terminology, $\cap h(e, A)$ is the set of innocently includable alternatives (II-alternatives), while $\cup h(e, A)$ is the set of alternatives that are not innocently excludable (non-IE-alternatives). If every non-IE-alternative is an II-alternative, it will hold that $\bigvee\big(\cup h(e, A) - \cap h(e, A)\big) = \bigvee\emptyset = \top$. Except for the case of plain disjunction which also instantiates (6) as shown above, all other cases discussed in Bar-Lev and Fox (2020) are similar to the case of free choice disjunctions in the sense that they involve non-IE-alternatives all of which are innocently includable. In other words, they are all cases where $\cup h(e, A) - \cap h(e, A) = \emptyset$ (cf. Bar-Lev and Fox 2020: 197, 202, 206, 210, 213–214, 216–217). We believe this holds generally in the literature (cf. e.g. Crnič 2019). We will discuss these cases in the talk.

5. Conceptual considerations – Bar-Lev and Fox (2020: 186) presents a possible underlying conception, which they call "cell identification," that has guided their thinking in proposing m as the meaning of $exh(A)(p)$.

(7) Exhaustifying p with respect to a set of alternatives C should get us as close as possible to a cell in the partition induced by C

The partition induced by the set of alternatives consititutes the question under discussion (Groenendijk and Stokhof 1984, Lewis 1988). Exhaustification is an attempt to get close to a complete answer. The more alternatives it assigns truth values to, the more cells are eliminated. Under this perspective, it is actually m', not m, which is a more natural candidate for $exh(A)(p)$, since m' eliminates not only cells where m is false but also cells where $\bigvee\big(\cup h(e, A) - \cap h(e, A)\big)$ is false, whereas m only eliminates cells where m is false. Another conceptual reason for prefering m' to m is that m' would make the derivation of strengthened meanings more similar to the derivation of modal domains, and thus would reveal the same mechanism being operative in two seemingly unrelated processes.

6. The nature of alternatives – Suppose we have $exh(A)(p)$ with the following properties: (i) $A = \{p, q, r, s, t\}$; (ii) $p \Rightarrow (r \vee s \vee t)$; (iii) $p \wedge \neg q \wedge r \wedge s \neq \bot$; (iv) $p \wedge \neg q \wedge r \wedge t \neq \bot$; (v) $(p \wedge \neg q \wedge r \wedge s) \wedge (p \wedge \neg q \wedge r \wedge t) = \bot$. Then we would have $m = p \wedge \neg q \wedge r$ and $m' = p \wedge \neg q \wedge r \wedge (s \vee t)$, and would be able to distinguish between the hypothesis that $exh(A)(p) = m$ and the hypothesis that $exh(A)(p) = m'$. It is not clear to us whether any theory of alternatives on the market would exclude this scenario. Given that its non-existence is either contingent or necessary, we should either look for a case which instantiates it or formulate the theory of alternatives in such a way as to rule it out.

Bar-Lev, M. & D. Fox. 2020. Free choice, simplification, and innocent inclusion. **Crnič, L. 2019.** Any, alternatives, and pruning. **Groenendijk, J. & M. Stokhof. 1984.** Studies on the Semantics of Questions and the Pragmatics of Answers. **Kratzer, A. 1981.** The notional category of modality. **Kratzer, A. 1991.** Modality. **Lewis, D. 1988.** Relevant implication.