

## Ignorance, Introspection, and Epistemic Modals

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**Abstract** Embedded epistemic modals are infelicitous under desire predicates when they are anchored to the belief state of the attitude holder (see, esp., [Anand & Hacquard 2013](#)). We present two ways of deriving this observation from an independently motivated property of desire predicates ([Heim 1992](#); [von Stechow 1999](#)).

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### 1 A puzzle about embedded epistemics

#### 1.1 The generalization

In (1-a), the epistemic modal **must** is anchored to what John knows, and in (1-b), to what the police knows.<sup>1</sup>

- (1) a. John believes that Mary, given what he knows, must be the murderer
- b. John believes that Mary, given what the police knows, must be the murderer

We say an epistemic modal embedded under an attitude verb is “subject oriented” when it is anchored to the belief state of the attitude holder. Now consider (2).

- (2) a. #John hopes that Mary, given what he knows, must be the murderer
- b. John hopes that Mary, given what the police knows, must be the murderer

When the embedding verb is the desire predicate **hope** instead of the doxastic predicate **believe**, the subject oriented reading of the embedded epistemic gives rise to deviance (cf. [Hacquard & Wellwood 2012](#); [Anand & Hacquard 2013](#)). Let us state the generalization.

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<sup>1</sup> For now, we remain vague about what “anchored” means. Here and in the rest of the paper, all embedded modals are intended to have the epistemic reading.

- (3) Embedded Epistemics Generalization (EEG)  
Subject oriented epistemics are infelicitous under desire predicates.

Note that without adverbials such as **given what the police knows**, embedded epistemics tend to have the subject oriented reading as default, hence the infelicity of the following sentences.<sup>2</sup>

- (4) a. #John hopes that Mary must be the murderer  
b. #John wants Mary to have to be the murderer  
c. #John demands that Mary must be the murderer

## 2 Ignorance

his paper presents a new derivation of the EEG. We start with the semantics of desire predicates, taking the verb **want** to be their representative. Heim (1992) argues for a non-monotonic semantics of **want** which says that

- (5) Heim’s analysis of **want**  

$$\llbracket \mathbf{want} \phi \rrbracket^{i,g}(x) = 1 \text{ iff}$$

$$\forall i' \in B_x^i (sim_{i'}(B_x^i \cap [\lambda i. \llbracket \phi \rrbracket^{i,g}]) <_{i,x} sim_{i'}(B_x^i \cap [\lambda i. \llbracket \neg \phi \rrbracket^{i,g}]))$$

where  $B_x^i$  is the set of indices compatible with  $x$ ’s beliefs at  $i$ ,  $sim_i(X)$  picks out those members of  $X$  that are “most similar” to  $i$ , and  $X <_{i,x} Y$  is the condition that every member of  $X$  is more desirable to  $x$  at  $i$  than any member of  $Y$ .<sup>3</sup> Thus, Heim’s analysis of **want** involves a comparison between the  $\phi$  and the  $\neg\phi$  alternatives within the attitude holder’s belief, stating, essentially, that for  $x$  to want  $\phi$  is for  $x$  to believe that every way of making  $\phi$  true is better than any way of making  $\neg\phi$  true.

In contrast to Heim, von Fintel (1999) argues for a monotonic, Kratzerian semantics of **want** which says that

- (6) von Fintel’s analysis of **want**  

$$\llbracket \mathbf{want} \phi \rrbracket^{i,g}(x) = 1 \text{ iff } max_{D_x^i}(B_x^i) \subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}]$$

where  $D_x^i$  is the set of  $x$ ’s desires at  $i$  and  $max_P(X)$  picks out those members of  $X$  that are “optimal” with respect to  $P$ .<sup>4</sup> According to this analysis, for  $x$  to want  $\phi$  is for  $x$  to find  $\phi$  true in the most desirable scenarios among those which he considers possible.

The explanation we are going to provide for the EEG does not require us to

<sup>2</sup> Because **want** does not take a tensed complement, the embedded modal cannot be **must**.

<sup>3</sup> To be explicit,  $X <_{i,x} Y$  iff  $\forall x \in X (\forall y \in Y (x <_{i,x} y))$ .

<sup>4</sup> Specifically,  $max_P(X) = \{i \in X \mid \neg \exists i' \in X (\{p \in P \mid p(i) = 1\} \subset \{p \in P \mid p(i') = 1\})\}$ . Note that  $D_x^i$ , in the lexical entry in (6), is a set of propositions, not a set of indices.

choose between (5) and (6). We can therefore remain agnostic with respect to the differences between the non-monotonic and the monotonic analysis of **want** as well as which of the two analyses may be more adequate. However, we will not remain agnostic with respect to what these analyses have in common. Both (5) and (6) make crucial reference to an information state, specifically the belief state of the attitude holder, and both turn out to predict that a **want**-sentence will be a triviality in case the subject believes the complement or its negation.<sup>5</sup> These predictions are intuitively false, since it does not seem valid to infer from **John believes that Mary smokes** or **John believes that Mary doesn't smoke** that **John wants Mary to smoke** is true, or is false. In fact, the last sentence is perceived as infelicitous if either of the first two sentences is true.

To resolve this problem, Heim and von Fintel both add a definedness condition to the semantics of **want**, given in (7), which requires that the subject of **want** not be opinionated, i.e. be ignorant, about its complement. As desired, this condition not only guarantees that we cannot infer from **John believes that Mary (doesn't) smokes** to **John wants Mary to smoke**, but also entails that these sentences are incompatible (see Heim 1992; von Fintel 1999 for discussion and refinements).

- (7) Ignorance  
 $\llbracket \text{want } \phi \rrbracket^{i,g}(x)$  is defined only if  $\exists i' \in B_x^i(\llbracket \phi \rrbracket^{i',g} = 1) \wedge \exists i' \in B_x^i(\llbracket \phi \rrbracket^{i',g} = 0)$

In what follows, we provide two accounts of the EEG, both of which capitalize on the Ignorance condition and which differ in their assumptions about modal semantics.<sup>6</sup>

### 3 Resolving the puzzle

In order to appreciate the import of Ignorance for the EEG, we need to make some assumptions about epistemic modals. We discuss two common approaches to them and show that, in concert with Ignorance, they allow for a routine explanation of the EEG.

<sup>5</sup> Consider Heim's analysis. If  $x$  believes  $\phi$ , then  $B_x^i \cap [\lambda i. \llbracket \neg \phi \rrbracket^{i,g}] = \emptyset$ , hence  $\text{sim}_{i'}(B_x^i \cap [\lambda i. \llbracket \neg \phi \rrbracket^{i,g}]) = \emptyset$ . Similarly, if  $x$  believes  $\neg \phi$ , then  $\text{sim}_{i'}(B_x^i \cap [\lambda i. \llbracket \phi \rrbracket^{i,g}]) = \emptyset$ . But given the definition of  $<_{i,x}$ , it is trivially true that  $X <_{i,x} Y$  if either  $X = \emptyset$  or  $Y = \emptyset$  (see note 3). Now consider von Fintel's analysis. Given that  $\text{max}_{D_x^i}(B_x^i) \subseteq B_x^i$  by definition, if  $x$  believes  $\phi$ , then  $B_x^i \subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}]$ , hence  $\text{max}_{D_x^i}(B_x^i) \subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}]$ , and if  $x$  believes  $\neg \phi$ , then  $B_x^i \subseteq [\lambda i. \llbracket \neg \phi \rrbracket^{i,g}]$ , hence  $\text{max}_{D_x^i}(B_x^i) \subseteq [\lambda i. \llbracket \neg \phi \rrbracket^{i,g}]$ , which means  $\text{max}_{D_x^i}(B_x^i) \not\subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}]$ .

<sup>6</sup> Note that Ignorance not only guarantees that we cannot infer from **John believes that Mary smokes** to **John wants Mary to smoke**, but also entails that these two sentences are incompatible.

### 3.1 Relational semantics

We start the standard relational semantics for epistemic modals in (8), where  $g(R)$  is the contextually determined accessibility relation.

- (8) Relational semantics of **must**  
 $\llbracket \mathbf{must}_R \phi \rrbracket^{i,g} = 1$  iff  $g(R)(i) \subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}]$

We assume that the subject oriented reading of embedded epistemics results from  $g(R)$  being set to  $[\lambda i. B_x^i]$ , where  $x$  is the subject of the embedding attitude verb. Now consider (9).

- (9) #John demands that Mary must be the murderer

Under the (infelicitous) subject oriented reading, Ignorance imposes on (9) the definedness condition in (10), with  $i$  the index of evaluation and  $\phi = \mathbf{Mary\ be\ the\ murderer}$ .

- (10)  $\exists i' \in B_j^i(\llbracket \mathbf{must}_R \phi \rrbracket^{i',g} = 1) \wedge \exists i' \in B_j^i(\llbracket \mathbf{must}_R \phi \rrbracket^{i',g} = 0)$ , i.e.  
 $\exists i' \in B_j^i(B_j^{i'} \subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}]) \wedge \exists i' \in B_j^i(B_j^{i'} \not\subseteq [\lambda i. \llbracket \phi \rrbracket^{i,g}])$

What (10) says is that John's belief does not rule out the possibility that he believes that Mary is the murderer, and does not rule out the possibility that he does not believe that Mary is the murderer. Thus, (10) says that John is ignorant about his own belief. Obviously, nothing we have said so far prevents (10) from being true. Thus, we predict, all things being equal, that (9) should be felicitous, contrary to observation.

There may be good reasons to believe that not all things are equal, however. One common hypothesis about epistemic agents is that they are in fact not ignorant about their own belief: if  $x$  believes  $p$  then  $x$  believes that  $x$  believes  $p$ , and if  $x$  does not believe  $p$  then  $x$  believes that  $x$  does not believe  $p$  (e.g., Hintikka 1962; Lewis 1969; Stalnaker 2002, among others).

- (11) Introspection  
 for all  $x, p, i$ , it holds that  $\forall i' \in B_x^i(B_x^{i'} \subseteq p) \vee \forall i' \in B_x^i(B_x^{i'} \not\subseteq p)$

With this additional hypothesis in hand, we can now explain the deviance of (9): Ignorance and Introspection, together, impose contradictory demands on the attitude holder of (9), since (10) is incompatible with (11). Note, importantly, that the conflict between Ignorance and Introspection arises only in the case of subject oriented epistemics under desire predicates: if the embedding verb is a doxastic predicate, Ignorance does not apply – this accounts for the acceptability of (1) –, and

if the reading is not subject oriented, Introspection is not problematic – this accounts for the acceptability of (2-b).

### 3.2 Domain semantics

Can we derive the deviance of (9) by formulating a different, non-relational, semantics for modals instead of adding Introspection to the theory? It turns out we can. Suppose we follow Yalcin (2008) and assume a “domain” semantics for modals, taking the index to be a pair  $\langle w, S \rangle$  with  $w$  a possible world and  $S$  an information state which is a set of possible worlds.<sup>7</sup>

$$(12) \quad \text{Domain semantics for **must**}$$

$$\llbracket \mathbf{must} \phi \rrbracket^{\langle w, S \rangle, g} = 1 \text{ iff } S \subseteq [\lambda w. \llbracket \phi \rrbracket^{\langle w, S \rangle, g}]$$

The subject oriented reading of an embedded epistemic modal, in this framework, will result from  $S$  being set to  $B_x^w$ , with  $x$  the subject of the embedding verb and  $w$  the world of evaluation for the attitude ascription. Suppose Ignorance is formulated as in (13),

$$(13) \quad \text{Ignorance (domain semantics version)}$$

$$\llbracket \mathbf{want} \phi \rrbracket^{\langle w, S \rangle, g}(x) \text{ is defined only if}$$

$$\exists w' \in B_x^w (\llbracket \phi \rrbracket^{\langle w', S \rangle, g} = 1) \wedge \exists w' \in B_x^w (\llbracket \phi \rrbracket^{\langle w', S \rangle, g} = 0)$$

the definedness condition imposed by Ignorance on (9), under the subject oriented reading, will then be (14), with  $\phi = \mathbf{Mary be the murderer}$  and  $w$  the world of evaluation.

$$(14) \quad \exists w' \in B_j^w (\llbracket \mathbf{must} \phi \rrbracket^{\langle w', B_j^w \rangle, g} = 1) \wedge \exists w' \in B_j^w (\llbracket \mathbf{must} \phi \rrbracket^{\langle w', B_j^w \rangle, g} = 0), \text{ i.e.}$$

$$\exists w' \in B_j^w (B_j^w \subseteq [\lambda w. \llbracket \phi \rrbracket^{\langle w, S \rangle, g}]) \wedge \exists w' \in B_j^w (B_j^w \not\subseteq [\lambda w. \llbracket \phi \rrbracket^{\langle w, S \rangle, g}]), \text{ i.e.}$$

$$B_j^w \subseteq [\lambda w. \llbracket \phi \rrbracket^{\langle w, S \rangle, g}] \wedge B_j^w \not\subseteq [\lambda w. \llbracket \phi \rrbracket^{\langle w, S \rangle, g}]$$

Of course, (14) is a contradiction. This means that we can say that the deviance of (9) is due to its having a non-satisfiable definedness condition. Note, again, that this situation arises only when the epistemic is embedded under a desire predicate and has the subject oriented reading. If the embedding verb is a doxastic predicate, Ignorance does not apply, and if the reading is not subject oriented, the existential quantification in (14) would not be superfluous. To illustrate the latter scenario, consider (15).

<sup>7</sup> Being true at an index will now mean being true with respect to  $w$ , as in the case of non-modalized sentences, or being true with respect to  $S$ , as in the case of modalized sentences. See Yalcin (2008) for more details.

(15) John demands that Mary must be the murderer according to the police

The complement of **demands** in (15), clearly, should be evaluated with respect to John's belief about what the police believes. Thus, Ignorance would impose on (15) the condition in (16), where  $B_p^{w'}$  is the police's belief at  $w'$ .

$$(16) \quad \exists w' \in B_j^w (B_p^{w'} \subseteq [\lambda w. \llbracket \phi \rrbracket^{(w,S),g}]) \wedge \exists w' \in B_j^w (B_p^{w'} \not\subseteq [\lambda w. \llbracket \phi \rrbracket^{(w,S),g}])$$

The existential quantifications in (16) are not superfluous, hence (16) is not contradictory.

### 3.3 Distinguishing between the two approaches?

The relational semantics account requires two definedness conditions, namely Ignorance and Introspection, while the domain semantics account requires only one, namely Ignorance. It follows, then, that an observation which requires abandoning at least one of these conditions will count as evidence in favor of domain semantics. With that said, consider (17).

(17) John wants to believe that Mary is the murderer

The data we have discussed until now involve epistemics embedded under desires and doxastic predicates. We have not looked at cases where a doxastic predicate is embedded under a desire predicate, which is what (17) is. It turns out that to the extent that (17) is felicitous, it supports the domain semantics account. Ignorance imposes on (17) the condition in (18-a), and Introspection imposes on it the condition in (18-b), with  $p =$  'that Mary is the murderer.'

$$(18) \quad \begin{array}{l} \text{a. } \neg(\text{John believes (he believes } p)) \wedge \neg(\text{John believes } \neg(\text{he believes } p)) \\ \text{b. } (\text{John believes (he believes } p)) \vee (\text{John believes } \neg(\text{he believes } p)) \end{array}$$

As (18-a) and (18-b) contradict each other, the felicity of (17), to the extent that it is real, means that one of these conditions is false, hence constitutes evidence in favor of domain semantics. It goes without saying that further investigation into the contrast between embedded attitude predicates and embedded modals, and into adjudicating between the relational and domain approach to epistemic modals, is mandated.

## 4 Residual issues

### 4.1 Existential modals

Ignorance and Introspection impose the same conditions on **must**  $\phi$  as they do on **might**  $\psi$ , where  $\psi$  is  $\neg\phi$ . Thus, we expect the observations we have made about **must** to hold for its dual **might** also. This is true to a large extent.

- (19) a. #John hopes that Mary, given what he knows, might be the murderer  
b. John hopes that Mary, given what the police knows, might be the murderer
- (20) a. John believes that Mary might be the murderer  
b. #John demands that Mary might be the murderer

However, there is a difference between **must** and **might**: it seems that in sentences without adverbials such as **given what x knows**, the non-subject oriented reading under **hope** is more easily obtained with **might** than with **must** (cf. Anand & Hacquard 2013).

- (21) a. #John hopes that Mary must be the murderer  
b. John hopes that Mary might be the murderer

We have no solution to this puzzle and will have to leave it to future work.

### 4.2 Variability with respect to Ignorance

We have formulated Ignorance as a felicity condition which is to be imposed on all desire predicates. Data show that reality is more fine-grained, and that Ignorance should likely be lexically constrained, with each verb, in principle, determining the relevant domain in its own way (cf., e.g., Heim 1992; von Stechow 1999; ?; Anand & Hacquard 2013, among others). Consider the contrast in (22).

- (22) a. I know Mary is playing video games but I want her to be swimming now  
b. #I know Mary is playing video games but I hope that she is swimming now
- (23) a. #I want to have been sick  
b. I wish to have been sick

Again, we have no solution to this puzzle and will leave it to future work.

### 4.3 Anand & Hacquard (2013)

In their pioneering work on the EEG, Anand & Hacquard (2013) adopt a preference-based, Heimian analysis of **want**, combining it with a domain semantics for modals, and with the assumption that the complement of **want** will be evaluated with respect to a “special” information state, namely  $\emptyset$ .<sup>8</sup>

$$(24) \quad \text{Anand and Hacquard's analysis of } \mathbf{want} \\ \llbracket \mathbf{want} \phi \rrbracket^{\langle w, S \rangle, g}(x) = 1 \text{ iff } [\lambda w'. \llbracket \phi \rrbracket^{\langle w', \emptyset \rangle, g}] <_{w, x} [\lambda w'. \llbracket \neg \phi \rrbracket^{\langle w', \emptyset \rangle, g}]$$

Coupled with the assumption that  $\llbracket \mathbf{must} \phi \rrbracket^{\langle w, S \rangle, g}$ , and  $\llbracket \mathbf{might} \phi \rrbracket^{\langle w, S \rangle, g}$ , are undefined if  $S = \emptyset$ , this semantics has the consequence that a **want** sentence is defined only when the complement of **want** is such that its evaluation makes no reference to the information state, i.e. that it is non-modalized.

In contrast to Anand and Hacquard’s account, our proposal is agnostic with respect to the assertive component of desire predicates as well as compatible with the tight connection between belief and desire argued for in Heim (1992) and von Fintel (1999). Furthermore, the proposal does not require any special assumptions about what information states epistemic modals can be relativized to in specific constructions. While there may well be strong reasons for adopting Anand and Hacquard’s treatment of **want** and their split between doxastic and desire predicates, the EEG does not necessarily furnish one.

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<sup>8</sup> The formulation in (24) renders Anand & Hacquard’s (36), i.e. their proposed semantics for **want**, in notation more consistent with that used in this paper, while keeping to the same content.



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