A more inclusive theory of numerals

Andreas Haida, ELSC & Tue Trinh, ZAS

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The cardinality view on numerical statements

- Standard analyses: numerical statements are predications of second order properties to concepts.

(1) John read 3 novels

- The truth condition of (1) is taken to be either (2-a) or (2-b).

(2) a. $|\{x \mid x \text{ is a novel} \wedge \text{John read } x\}| = 3$
    b. $|\{x \mid x \text{ is a novel} \wedge \text{John read } x\}| \geq 3$

Identifying a problem for the cardinality view

- Extending the traditional analysis to (3) yields absurdity.

(3) John read 2.5 novels

(4) a. $|\{x \mid x \text{ is a novel that John read}\}| = 2.5 \Leftrightarrow \bot$

b. $|\{x \mid x \text{ is a novel that John read}\}| \geq 2.5 \Leftrightarrow$
$|\{x \mid x \text{ is a novel that John read}\}| \geq 3$

- Suppose John read *Brothers Karamazov*, *Crime and Punishment*, one-half of *Demons*, and nothing else.

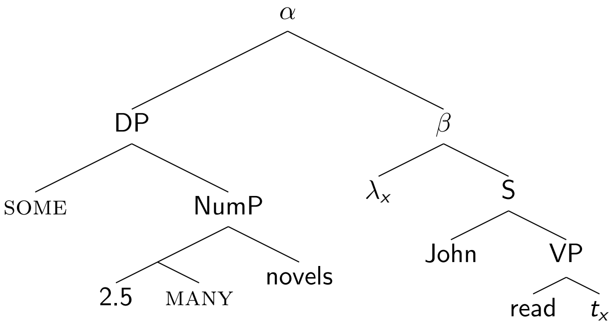
(5) a. John read 2.5 novels TRUE

b. John read 3 novels FALSE

Our proposal for decimal statements

- The logical form of **John read 2.5 novels** is (6), where SOME and MANY are covert (cf. Hackl, 2000).

(6)



- Goal: formulating a semantics for MANY.

The semantics of MANY

- Plural nouns denote cumulative predicates, i.e. subsets of $\mathcal{D}_e$ which are closed under $\sqcup$ (cf., e.g., Chierchia, 1998).
- For each predicate P, the set of P atoms, P_{at}, is defined as

(7) $P_{at} := \{x \in P \mid \neg \exists y \,.\, y \sqsubset x \wedge y \in P\}$.

Let b = *Brothers Karamazov* and c = *Crime and Punishment*.

  - $b \sqcup c \notin [\![\textbf{novels}]\!]_{at}$ since $b \sqcup c$ has proper parts that are novels.
  - $b, c \in [\![\textbf{novels}]\!]_{at}$ since neither b nor c has proper parts that are novels.

- The semantics we propose for MANY is (8), where d ranges over degrees.

(8) $[\![\text{MANY}]\!](d)(P) = [\lambda x \in \mathcal{D}_e \,.\, \mu_P(x) \geq d]$

- Thus, **John read 2.5 novels** is true iff there exists an individual x such that $\mu_{[\![\textbf{novels}]\!]}(x) \geq 2.5$ and John read x.

The definition of the term $\mu_{[\![\textbf{novels}]\!]}(x)$

- $\mu_{[\![\textbf{novels}]\!]}(x)$ represents "how many novels are in x."
- Goal: to be able to count novels in such a way that proper subparts of novels, which are not novels, also contribute to the count.
- To this end, we propose to explicate the measure function μ_P as follows.

$$\mu_P(x) = \begin{cases} \mu_P(y) + 1 & \text{if } a \sqsubset x,\ y \sqcup a = x, \text{ and } y \sqcap a = \bot \text{ for some } a \in P_{at} \\ \mu_a(x) & \text{if } x \sqsubseteq a \text{ for some } a \in P_{at} \\ \# & \text{otherwise} \end{cases}$$

- Thus, each P atom which is a subpart of x adds 1 to $\mu_P(x)$.
- If x is an P atom or a subpart of a P atom a, $\mu_P(x)$ is $\mu_a(x)$.

The characterization of the measure function μ_a

- $\mu_a(x)$ represents "how much of the P atom a is in x."
- The measure function μ_a is explicated as follows.

(9) For each $a \in P_{at}$,

a. μ_a maps $\{x \in \mathcal{D}_e \mid x \sqsubseteq a\}$ onto $(0, 1] \cap \mathbb{Q}$

b. $\mu_a(x \sqcup y) = \mu_a(x) + \mu_a(y)$ for all $x, y \in \text{dom}(\mu_a)$ such that $x \sqcap y = \bot$

c. $\mu_a(a) = 1$

Prediction 1

- (10-a) is neither contradictory nor equivalent to (10-b).

(10) a. John read 2.5 novels
b. John read 3 novels

- Our explanation: $\mu_{[\![\textbf{novels}]\!]}(x) \geq 2.5$ is neither contradictory nor equivalent to $\mu_{[\![\textbf{novels}]\!]}(x) \geq 3$:

  Let b, c, d be novels, and d' be one-half of d.

  - $\mu_{[\![\textbf{novels}]\!]}(d') = \mu_d(d') = 0.5$
  - $$\begin{aligned} \mu_{[\![\textbf{novels}]\!]}(b \sqcup c \sqcup d') &= \mu_{[\![\textbf{novels}]\!]}(c \sqcup d') + 1 \\ &= \mu_{[\![\textbf{novels}]\!]}(d') + 1 + 1 \\ &= \mu_d(d') + 1 + 1 \\ &= 0.5 + 1 + 1 \\ &= 2.5 \end{aligned}$$
  - Thus, $\mu_{[\![\textbf{novels}]\!]}(x) \geq 2.5$ is not contradictory.
  - The non-equivalence follows from the logical truth that $2.5 < 3$ and the fact that there is an x such that $\mu_{[\![\textbf{novels}]\!]}(x) = 2.5$.

Prediction 2: Non-additivity of measures of fractions

- We predict that (11) is valid and (12) invalid.

(11) Ann read 1 Russian novel
Ann read 1 French novel
$\vDash$ Ann read 2 novels

(12) Ann read 0.75 Russian novels
Ann read 0.75 French novels
$\nvDash$ Ann read 1.5 novels (cf. Liebesman, 2016)

- Let b be the Russian novel and c the French novel of the premises of (11). As $\mu_{[\![\textbf{novels}]\!]}(b \sqcup c) = 2$, the conclusion follows.
- On the other hand, let b' and c' be the fractional counterparts of b and c of the premises of (12).
- There is no $a \in [\![\textbf{novels}]\!]_{at}$ such that $a \sqsubset b' \sqcup c'$ or $b' \sqcup c' \sqsubseteq a$, which means $\mu_{[\![\textbf{novels}]\!]}(b' \sqcup c') = \#$, which means $\mu_{[\![\textbf{novels}]\!]}(b' \sqcup c') \ngeq 1.5$.
- This means the conclusion of (12) doesn't follow.

Prediction 3: Non-monotonicity of MANY

- We predict that the scale provided by MANY cannot serve as scale of comparison.
- That is, we do *not* wrongly predict that the argument in (13) is valid.

(13) John read 2.5 novels
Mary read 2 novels
$\nvDash$ John read more novels than Mary

- As just seen, the scale $[\lambda x \lambda d.\, [\![\text{MANY}]\!](d)([\![\textbf{novels}]\!])(x)]$ is non-monotonic:

$$[\lambda x \lambda d.\, [\![\text{MANY}]\!](d)([\![\textbf{novels}]\!])(x)](b')(0.75) = 1$$
$$[\lambda x \lambda d.\, [\![\text{MANY}]\!](d)([\![\textbf{novels}]\!])(x)](b' \sqcup c')(0.75) \neq 1$$

- However, scales of comparison must be monotonic (Wellwood et al. 2012):

(14) John ate 90 degree hot spaghetti
Mary 70 degree hot spaghetti
$\nvDash$ John ate more spaghetti than Mary

Prediction 3 – cont'ed

- We note that the argument in (15) is valid.

(15) John read 3.5 novels
Mary read 2 novels
$\vDash$ John read more novels than Mary

- To account for this fact, we tentatively assume that measurement can be restricted to atoms and sums of atoms.
- This means to say that the relevant scale of comparison is the monotonic scale in (16).

(16) $[\lambda x \lambda d. [\![\textsc{many}]\!](d)([\![\textbf{novels}]\!])(x \sqcap \bigsqcup [\![\textbf{novels}]\!]_{at})]$

Prediction 4: Deviant decimal statements

- We predict that (17) is deviant.

(17) #John read 0.5 quantities of literature

- According to our semantics of MANY, (17) entails the existence of an individual x such that $\mu_{[\![\textbf{qol}]\!]}(x) \geq 0.5$.
- This, in turn, entails the existence of some $a \in [\![\textbf{qol}]\!]_{at}$ such that $x \sqsubseteq a$.
- Given that any subpart of a quantity of literature is itself a quantity of literature, we have $[\![\textbf{qol}]\!]_{at} = \{x \in [\![\textbf{qol}]\!] \mid \neg\exists y \sqsubset x \wedge y \in [\![\textbf{qol}]\!]\} = \varnothing$.
- Thus, there is no $a \in [\![\textbf{qol}]\!]_{at}$, which means there is no x such that $\mu_{[\![\textbf{qol}]\!]}(x) \geq 0.5$.
- This means (17) is false. Furthermore, it is analytically false, which is to say false by virtue of the meaning of the word **quantity**.
- This, we hypothesize, is the reason for its being perceived as deviant.

Prediction 5: No numerical gaps

- We predict (18), which we claim to be a fact about natural language.

(18) There is no numerical gap in the scale which underlies measurement in natural language

- That is, to the extent **John read 2.5 novels** is meaningful, **John read 2.55 novels** is too, as well as **John read 2.555 novels**, or any member of $\{$**John read n novels** $\mid [\![\mathbf{n}]\!] \in \mathbb{Q}^+\}$.
  - Since $0.5, 0.55, 0.555, \ldots \in \mathrm{ran}(\mu_a)$, for all $a \in [\![\mathbf{novels}]\!]_{at}$.
  - Since, by stipulation, μ_a is a function onto $(0,1] \cap \mathbb{Q}$.
- Note, importantly, that we cannot guarantee (18) by stipulating the UDM (Fox & Hackl 2006):

(19) $S := \mathbb{Q}^+ \backslash \{x \in \mathbb{Q} \mid 3 < x \leq 4\}$

S is a dense scale. However, S contains a gap.

- The UDM, therefore, does not guarantee that **John read 3.5 novels** is meaningful.

Controversial data: sums that measure less than 1

- For (21), we make different predictions from Liebesman.

(20) Ann read 0.75 Russian novels
Ann read 0.75 French novels
$\not\models$ Ann read 1.5 novels

(21) Ann read 1 German novel
Ann read 0.25 Russian novels
Ann read 0.25 French novels
$\not\models$ Ann read 1.5 novels (us)
$\models$ Ann read 1.5 novels (Liebesman 2016)

- Liebesman (2016) takes the validity of (21) to be an empirical fact and captures the (alleged) contrast between (20) and (21) by stipulation.
- Our recursive definition of μ_A predicts no contrast, and we are not sure about (21).
- If Liebesman's empirical claim is correct, there's no easy fix for our approach.

Open question 1: Unfinished objects of creation

- Our semantics makes the wrong prediction that (22) is false in the actual world.

(22) The *Unvollendete* is 0.5 symphonies

  - Let u be the *Unvollendete*.
  - In the actual world, the there is no $a \in [\![\textbf{symphonies}]\!]_{at}$ such that $u \sqsubseteq a$.
  - Since our semantics is extensional, it follows that $\mu_{[\![\textbf{symphonies}]\!]}(u) \neq 0.5$.

- Obviously, modality is involved: while there is no symphony s such that $\mu_s(u) = 0.5$, there could be one, since the last two movements could have been completed.
- Thus, counting symphonies seems to be about what could be a symphony, not what is actually a symphony.

Open question 1 – cont'ed

- This means we should, perhaps, revise our semantics so as to predict that to be half an P is to be half of something which is an P atom in some possible world.
- However, we do not want to predict, incorrectly, that (23) is true, for example.

(23) *Crime and Punishment* is 0.5 symphonies

- Thus, while there certainly is a possible world w in which the entity that is *Crime and Punishment* in the world of evaluation is a subpart of a symphony, we want w to be inaccessible from the world of evaluation.
- Plausibly, specifying the relevant accessibility relation in this particular case amounts to fleshing out the concept of "symphony," and specifying it in the general case, to fleshing out the concept of "concept." We leave this task to future work.

The Bible problem

- Both of the following two statements are true.

(24) a. The Torah is 0.20833 of the Hebrew Bible
b. The Torah is 0.10869 of the Christian Bible

- If I read the Torah, how many bibles did I read?
- We run the risk that our function $\mu_{[\![\textbf{bibles}]\!]}$ is not well defined since there are two atoms that the Torah is part of.
- Our solution: the same text can be two different objects depending on what it is part of.

Conclusion

- The cardinality theory of numeral statements cannot be extended to decimal statements.
- The measurement function we propose in place of the cardinality function is non-monotonic and continuous.
- This accounts for a number of intuitions about the logical relations between decimal statements.
- An intensionalized version of our analysis seems desirable.

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