A case for no Ks

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Abstract. We present a novel observation about modified numerals and discuss how it may pose a problem for the syntactic representation of speaker's belief, hence the grammatical derivation of ignorance inferences.

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1. Two ways of deriving the ignorance inference of "at least"

It has been observed that the modifier *at least* gives rise to "ignorance inferences." For example, (1) implies that the speaker is not sure whether there are exactly two students (Geurts and Nouwen, 2007; Buring, 2008; Schwarz, 2016).[1]

(1) there are at least two students in the classroom
$\rightsquigarrow$ $\neg K(\textit{exactly two}) \wedge \neg K\neg(\textit{exactly two})$
"the speaker is not sure whether there are exactly two students in the classroom"

This inference can be explained in terms of "exhaustification." One implementation of this explanation assumes (2), where the silent operator exh_A is defined as in (3) (cf. Fox, 2007a, b; Chierchia et al., 2012).

(2) Syntactic assumption
Every sentence ϕ can be parsed as $exh_A\ \phi$

(3) Definitions
a. $exh_A\ \phi \Leftrightarrow \phi \wedge \bigwedge\{\neg\psi : \psi \in IE(\phi, A)\}$
"$exh_A\ \phi$ is true iff ϕ is true and every ψ which is innocently excludable given ϕ and A is false"
b. $\psi \in IE(\phi, A)$ iff $\psi \in \bigcap\{A' \mid A'$ is a maximal subset of A s.t. $\{\phi\} \cup \{\neg\phi' \mid \phi' \in A'\}$ is consistent$\}$
"ψ is innocently excludable given ϕ and A iff ψ is a member of A and $\phi \wedge \neg\psi$ does not entail any disjunction of members of A not entailed by ϕ"

The idea is that by uttering ϕ, the speaker is actually saying $exh_A\ \phi$, which settles not just ϕ but also certain members of the set A, i.e. the set of "alternatives" of ϕ.[2] For present purposes, we need not go into details of how A is determined. It suffices to note a fact about A which, to the best of our knowledge, underlies every conception of this set which has been proposed in the literature. It is this.

(4) Fact about A
If $exh_A\ \phi$ is relevant, every member of A is relevant

From the Gricean maxims of Quality, Quantity, and Relation, it follows that what a speaker says is relevant and settles all relevant propositions he is not ignorant about about (cf. Grice,

[1] We will write "$K\phi$" to represent the fact that ϕ is entailed by the speaker's belief, which means the speaker's ignorance about ϕ can be represented as "$\neg K\phi \wedge \neg K\neg\phi$."
[2] A sentence ϕ settles another sentence ψ iff ϕ either entails ψ or entails $\neg\psi$.

1967). Given (4), this means the following (Kroch, 1972; Fox, 2007a, b; Chierchia et al., 2012).

(5) Fact about $exh_A\ \phi$
The speaker of $exh_A\ \phi$ is ignorant about members of A which are not settled by $exh_A\ \phi$

Let us look at an example. Suppose someone utters "John talked to Mary or Sue." By virtue of (2), what he is saying is not the simple disjunction $m \vee s$, but its exhaustification $exh_A(m \vee s)$. Under the assumption that A, in this case, is the set in (6a) (cf. Sauerland, 2004), we derive (6b) and (6c).

(6) $exh_A(m \vee s)$ 'John talked to Mary or Sue'
a. $A = \{m \vee s, m, s, m \wedge s\}$
b. $exh_A(m \vee s) \Leftrightarrow (m \vee s) \wedge \neg(m \wedge s)$
c. $exh_A(m \vee s) \rightsquigarrow \neg Km \wedge \neg K\neg m \wedge \neg Ks \wedge \neg K\neg s$

The equivalence in (6b) follows from (3): $exh_A(m \vee s)$ entails $(m \vee s)$ because $(m \vee s)$ is the sister of exh_A, and entails $\neg(m \wedge s)$ because $(m \wedge s)$ is innocently excludable given $(m \vee s)$ and A. The other two members of A, m and s, are not settled by $exh_A(m \vee s)$. From (5), then, we derive (6c). Thus, we expect the speaker who utters "John talked to Mary or Sue" to be saying that John did not talk to both Mary and Sue, and to be ignorant, i.e. unsure, about whether John talked to Mary as well as about whether John talked to Sue. It is generally agreed that this aligns with the attested intuition.

A crucial ingredient in the account, obviously, is (6a). Predictions about what inferences a sentence licenses vary according to what its alternatives are assumed to be and can, therefore, be used to measure the success of theories on alternatives. Coming back to the case of modified numerals, we will assume, for present purposes, that (7) holds, i.e. that alternatives of (8a) are (8b) and (8c), which are generated by replacing *at least* with *exactly* and *more than* (cf. Kennedy, 2015; Buccola and Haida, 2018).

(7) Scale mates of *at least n*
at least n alternates with *exactly n* and *more than n*

(8) a. there are at least two student in the classroom
b. there are exactly two students in the classroom
c. there are more than two students in the classroom

We are now in the position to account for the ignorance inference licensed by (8a) which is discussed at the beginning of this paper. In fact, there are two ways to derive this inference. One, call it the "pragmatic" derivation, proceeds as follows. By virtue of (2), the speaker who utters (8a) is saying (9a). Given (7), the set of alternatives is (9b). Given (3), $exh_A(\textit{at least two})$ settles neither *exactly two* nor *more than two*. It then follows, from (5), that the speaker who utters (8a), thereby saying (9a), is ignorant about *exactly two* and *more than two*, which means, equivalently, that he is ignorant about *exactly two*. This results aligns with intuition.

(9) a. $exh_A(\textit{at least two})$ 'there are at least two student in the classroom'
b. $A = \{\textit{at least two}, \textit{exactly two}, \textit{more than two}\}$
c. $exh_A(\textit{at least two}) \rightsquigarrow \neg K(\textit{exactly two}) \wedge \neg K\neg(\textit{exactly two})$

Another way to derive the ignorance inference of *at least*, call it the "semantic derivation," proceeds as follows. First, we will assume that the speaker's belief is explicitly represented in the syntax (cf. Meyer, 2014; Buccola and Haida, 2018).

(10) Syntactic assumption
The lexicon contains an operator, K, which means 'the speaker believes that' and which can be appended to every sentence.[3]

This assumption, in conjunction with (2), allows us to postulate (11b) as what is said by the speaker who utters (11a). From (7), it follows that the set of alternatives is (11c). By virtue of (3), both $K(\textit{exactly two})$ and $K(\textit{more than two})$ are innocently excludable. This means that (11b) is equivalent to (11d), which is in turn equivalent to (11e).

(11) a. there are at least two students in the classroom
b. $exh_A(K(\textit{at least two}))$
c. $A = \{K(\textit{at least two}), K(\textit{exactly two}), K(\textit{more than two})\}$
d. $K(\textit{at least two}) \wedge \neg K(\textit{exactly two}) \wedge \neg K(\textit{more than two})$
e. $K(\textit{at least two}) \wedge \neg K(\textit{exactly two}) \wedge \neg K \neg(\textit{exactly two})$

As we can see, the ignorance inference of *at least*, in the semantic derivation, becomes part of the literal meaning of what is said. In the pragmatic derivation, on the other hand, it arises from (5), which is a consequence of Gricean maxims.

Which derivation is correct? This question, in principle, is an empirical one: there is no a priori reason to assume that linguistic facts do not exist which favor settling it one way or another. We will argue that the contrast in (12) is such a fact.

(12) a. there are at least two students in the classroom
b. *there are at least zero students in the classroom

The numeral *zero* cannot be modified by the adverb *at least*. We will propose an account of this observation which makes a case that speakers' belief is not explicitly represented, i.e. "a case for no *K*s" in the syntax. To the extent that our account is correct, it is the pragmatic derivation of the ignorance inference of *at least* which is correct, as the semantic derivation crucially requires K to be syntactically represented.

The presentation of our account requires laying some groundwork. This task is undertaken in the next section.

2. L-Analyticity and the theory of *zero*

It has been claimed that a sentence can be deviant if it is tautological or contradictory purely by virtue of the configuration of logical constants in it (Barwise and Cooper, 1981; Fintel, 1993; Gajewski, 2003; Chierchia, 2006; Abrusán, 2007; Gajewski, 2009; Abrusán, 2011). As an example, consider the contrast in (13), discussed in Fintel (1993).

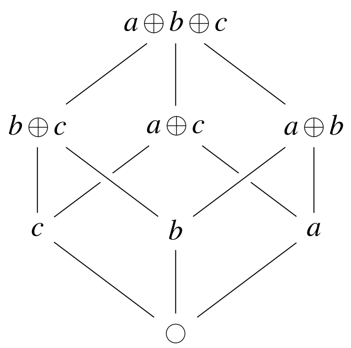
(13) a. everyone but Bill danced
'everyone who is not Bill danced & ¬everyone danced'
b. *someone but Bill danced
'someone who is not Bill danced & ¬someone danced'

[3]For concreteness, let us say that K has the following semantics: $[\![K\ \phi]\!]^w = 1$ iff $[\![\phi]\!]^{w'} = 1$ for every world w' compatible with what the speaker believes in w.

Under the sentences we give very informal paraphrases of von Fintel's semantic analyses which, nevertheless, suffice to show that (13a) is not, while (13b) is, a contradiction. Moreover, any replacement of the non-logical words in (13b), which are *one*, *Bill* and *danced*, would still result in a contradiction. It is in this sense that the sentence is said to be "L-analytical."[4] We will write "$\phi \Leftrightarrow_L \top$" or "$\phi \Leftrightarrow_L \bot$" to say that ϕ is L-analytically tautological or L-analytically contradictory, respectively.

L-analyticity is crucially appealed to by the theory of the numeral *zero* which we will assume here. This is the theory proposed in Bylinina and Nouwen (2018), according to which every plural noun has in its denotation a special element, $\bigcirc$, whose atoms count 0. To illustrate, suppose *a*, *b* and *c* are the only students in the world. The extension of the plural noun *students* would the be set containing all elements in the complete lattice below.

(14) $[\![\text{students}]\!] = \{a, b, c, a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c, \bigcirc\}$



The phrase *2 students*, for example, would denote the set of $[\lambda x[x \in [\![\text{students}]\!] \wedge \#x = 2]]$, i.e. the set $\{a \oplus b, b \oplus c, a \oplus c\}$.[5] We will assume that the existential sentence *there are 2 students* amounts to the claim that this set is not empty.

(15) there are 2 students $\Leftrightarrow [\exists x[x \in [\![\text{students}]\!] \wedge \#x = 2]]$

What about sentences with the numeral 0, for example (16)?

(16) there are zero students

Intuitively, (16) is well-formed. Now suppose (16) is parsed as (17), yielding the meaning 'there are zero or more students,' we will have an L-analytical sentence, and thus make the false prediction that (16) is deviant.

(17) $[_S$ there are zero students$]$
$\Leftrightarrow \exists x(\#x = 0 \wedge x \in [\![\text{students}]\!]) \Leftrightarrow_L \top$

However, parsing (16) with exh_A as in (18), yielding the meaning 'there are zero and no more students,' will rescue it from being an L-analytical sentence.[6]

(18) $[_S$ exh_A $[_S$ there are zero students$]]$
$\Leftrightarrow \exists x(\#x = 0 \wedge x \in [\![\text{students}]\!]) \wedge \neg \exists x(\#x > 0 \wedge x \in [\![\text{students}]\!]) \not\Leftrightarrow_L \top$

[4] This is also a simplification. For the full-fledged definition of L-analyticity, see Gajewski (2003, 2009).
[5] The measure function # maps an individual to its atom count, i.e. the number of atoms it contains.
[6] We assume that **zero** is a numeral and thus alternates with other numerals.

Consequently, *zero* always means 'zero and no more.' See Bylinina and Nouwen (2018) for arguments that this is in fact the case.

3. Settling the issue

Let us now come back to the novel observation mentioned at the end of section 1, namely that *zero* cannot be modified by *at least*.

(19) a. there are at least two students in the classroom
 b. *there are at least zero students in the classroom

To give empirical support to our factual claim, we conducted an experiment on Amazon MTurk, whose results are shown in the figures below.

Figure 1: Boxplot of *at least 2* and *at least 0*

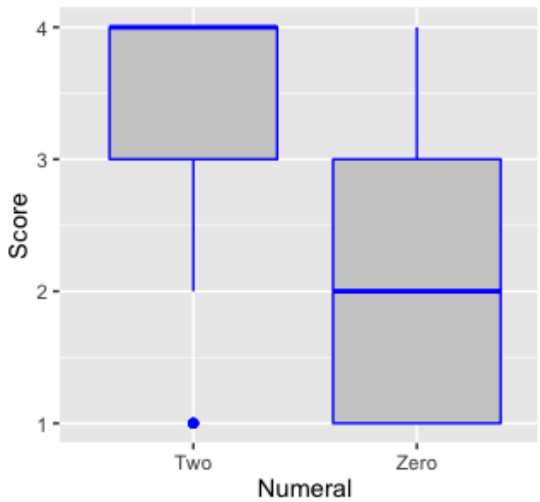
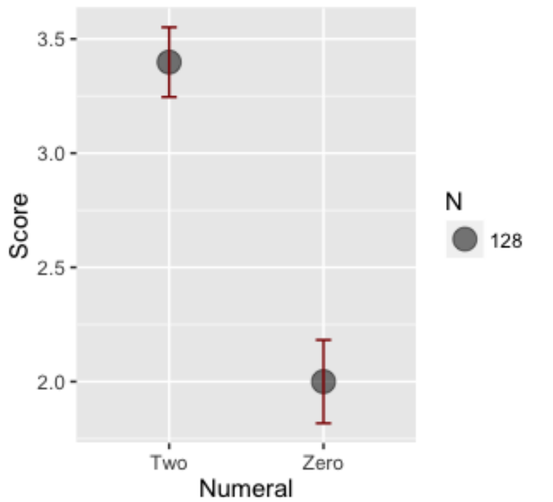


Figure 2: Means of *at least 2* and *at least 0*



As we can see, sentences with *at least two* receive a much higher score, i.e. are much more acceptable, than those containing *at least zero*.

We are now in the position to settle the question which of the two derivations of the ignorance inference of *at least* is correct. Recall that the semantic derivation crucially depends on the assumption that K, the operator representing speaker's belief, is explicitly represented. Suppose, then, that this is the case, i.e. that K is explicitly represented. The sentence **there are at least zero students* will then have (20) as a possible parse.

(20) $exh_A(K(\text{there are at least zero students in the classroom}))$

Given that *at least* alternate with *exactly* and *more than*, the set of alternative A would contain (21a) and (21b).

(21) a. $K(\text{there are exactly zero students in the classroom})$
 b. $K(\text{there are more than zero students in the classroom})$

This means that (20) is not analytical, hence not L-analytical, as it means (22), which is true iff the speaker's belief contains worlds where there are no students in the classroom and worlds where there are some students in the classroom, i.e. iff the speaker doesn't know whether there are any students in the classroom at all.

(22) K(there are at least zero students in the classroom) $\wedge \neg K$(there are exactly zero students in the classroom) $\wedge \neg K$(there are more than zero students in the classroom)

Now suppose K is not syntactically represented. Then, **there are at least zero students in the classroom* has (23) as available parse.

(23) exh_A(there are at least zero students in the classroom).

In this case, the set of alternatives A would contain (24a) and (24b), none of which is innocently excludable.

(24) a. there are exactly zero students in the classroom
b. there are more than zero students in the classroom

This means that (23) is analytical. Furthermore, it is L-analytical, assuming that *at least* is a logical term, i.e. one whose meaning is invariant across possible worlds. In fact, (23) ends up having the same meaning which *there are zero students* would have if it is not exhaustified: the adverb *at least* makes exhaustification vacuous.

We have considered two options: (i) K is explicitly represented in the syntax, and (ii) K is not so represented. We have seen that the first option predicts **there are at least zero students in the classroom* to have a parse which is not L-analytical, while the second option predicts this sentence to be L-analytical. Given that the sentence is perceived to be deviant, we have an argument for the second option and against the first. Since the semantic derivation of the ignorance of *at least* presupposes that the first option is available, we have an argument that that derivation is wrong, hence that the pragmatic derivation is correct.

4. Residual issues

4.1. A prediction

We predict that the *meaning* of (19b) can be felicitously expressed by a non-L-analytical sentence, such as (25a), whose LF is (25b) (Hurford, 1974; Chierchia et al., 2012; Fox and Spector, 2018).

(25) a. there are zero or more students
b. exh_A(there are zero students) or (there are more than zero students)

A Google search of, e.g., the phrase **0 or more times** gives $170,000$ results, while **at least 0 times** only gives $2,780$ results.

4.2. Two-sided meaning for numerals

One argument that **zero** is a numeral, not a quantifier, is that it is neither downward entailing nor does it have the distribution of a generalized quantifier (Nouwen & Bylinina's 2017):

(26) a. no/*zero students said anything
b. the number of students in the classroom is zero/*no

Suppose numerals have a two-sided meaning as a matter of semantic content, as proposed by several works (Breheny, 2008; Geurts, 2006; Kennedy, 2015). We will correctly derive that *there are zero students* is non-tautological, and that *there are at least zero students* is L-tautological.

(27) a. there are zero students $\Leftrightarrow$ exh_C(there are zero students)
$\Leftrightarrow \max\{n \mid \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]\} = 0$
$\Leftrightarrow$ exh_C [there are at least 0 students] $\not\Leftrightarrow \top$ (where, like before, $\bigcirc \in [\![\text{students}]\!]$ and $\#\bigcirc = 0$)

b. there are at least 0 students $\Leftrightarrow$ exh_C(there are at least 0 students)
$\Leftrightarrow \max\{n \mid \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]\} \geq 0$
$\Leftrightarrow_L \top$

However, we still derive, incorrectly, that the deviance of *at least zero* is obviated under universal quantification:

(28) $\text{exh}_C\,(K(\text{there are at least 0 student}))$
$\Leftrightarrow K(\max\{n \mid \exists y[y \in [\![\text{students}]\!] \wedge \#y = n]\} \geq 0])$
$\wedge \neg K(\max\{n \mid \exists y[y \in [\![\text{students}]\!] \wedge \#y = n]\} = 0])$
$\wedge \neg K(\max\{n \mid \exists y[y \in [\![\text{students}]\!] \wedge \#y = n]\} > 0])$
$\not\Leftrightarrow \top$

Thus, assuming the two-sided meaning for numerals will not rescue the semantic derivation of the ignorance inferences of *at least*.

4.3. The logical status of scales

We have given a semantic explanation for the incompatibility of *at least* and *zero*. Our account, thus, would be corroborated by facts which suggest that this incompatibility is not morphological. We believe the following contrast is such a fact.

(29) a. The temperature is at least zero degrees Celsius
b. #The temperature is at least zero degrees Kelvin

The contrast shows that it is not the morphological word *zero* which resists combination with *at least*, but the meaning of this word: *zero* in *zero degrees Celsius* does not denote the lowest point of the scale, hence does not really mean 'zero.' This is different with *zero* in *zero degrees Kelvin*, which denotes absolute zero and hence the lowest point of the relevant scale.

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