Notes on counting and L-analyticity

We present quantitative data regarding some novel observations about the numeral **zero**. We propose a tentative account of these observations, and discuss the implications it has for existing theories of exhaustification and L-analyticity, and the semantics of **zero**. We start by noting that **zero** cannot be modified by the superlative quantifier **at least** if it refers to the scalar endpoint 0, see (1a) v (1b).

(1) a. there are at least two students in the classroom
b. *there are at least zero students in the classroom

To support the empirical claim in (1), we conducted an experiment on Amazon MTurk. 32 English speakers rated the naturalness of 4 sentences like (1a) and (1b) on a 4-point scale. Sentences with **at least two** received the highest score 4 ('natural') by $\geq 50\%$ of all subjects, while sentences with **at least zero** received the two lowest scores 2 and 1 ('weird') by $\geq 50\%$ of all subjects. The difference in the means of the scores (3.4 v 2.0) is highly significant ($p < 2.2^{-16}$).

To derive the deviance of (1b), we assume for exposition that $[\![\textbf{at least } \boldsymbol{n}]\!](P)(Q) = 1$ iff $|P \cap Q| \geq n$, where $\boldsymbol{n}$ is a bare numeral and n the scale point that it refers to. Moreover, we follow Fox & Hackl (2006) in assuming that (modified) numerals belong to the 'logical' vocabulary of natural languages. Then, (1b) is analytically true and, moreover, L-analytically true (henceforth written as "(1b) $\Leftrightarrow_L \top$"), since **at least zero** belongs to the 'logical skeleton' of (1b) and denotes a constant function with value 1 (truth). Therefore, (1b) is ungrammatical (Gajewski 2003).

Implications for the theory of exhaustification and L-analyticity: We explore the potential consequences of exhaustifying L-analytically true expressions, and conclude that exhaustification cannot obviate ungrammaticality induced by L-analyticity. First, we observe that exhaustification of (1b) is semantically inconsequential. That is, if $\textbf{exh}_C$ is an exhaustifying operator that respects innocent excludability (Fox 2007), then the structure in (2a), which contains $\textbf{exh}_C$, has the same truth condition as the corresponding structure without $\textbf{exh}_C$, see (2c), given that the domain C is the set in (2b) (or any other set containing only alternatives that contradict each other). Thus, (1b) does not provide a testing ground for our exploration.

(2) *there are at least zero students (in the classroom)
a. $[_\psi$ exh$_C$ $[_\phi$ there are at least zero students$]]$
b. $C = \{\underbrace{\text{there are more than zero students}}_{\text{non-excludable}}, \underbrace{\text{there are exactly zero students}}_{\text{non-excludable}}\}$
c. $\psi \Leftrightarrow \phi \Leftrightarrow_L \top$

However, we observe that deviance persists with embedding of **at least zero** under a universal quantifier, see (3a) and (3b). To support these judgments, we conducted an experiment on Amazon MTurk. Specifically, we tested the claim that **every ... at least zero** has a different status from **every ... zero or more**, **every ... at least two**, and **every ... two or more** (157 subjects giving 1 'weird'/'not weird' judgment per sentence type). The proportion of 'weird' responses to **every ... at least zero** is greater than that to its **zero or more** counterpart (40% and 28%, respectively, $p = 0.01605$). In contrast, the proportions of 'weird' responses to **every ... at least two** and **every ... two or more** are equal (7% and 12%, respectively, $p = 0.34$) and smaller from **every ... at least zero** and **every ... zero or more**.

(3) a. *every human has at least zero children
b. *you are required to read at least zero books

Importantly, if exhaustification could obviate ungrammaticality (3a) and (3b) would be expected to be non-deviant. Here is why: given that (3b) has the parse in (4a), it has the non-tautological truth condition in (4c) (expressing free choice to not read a book and to read a book), since the alternatives in C do not contradict each other and are hence innocently excludable, see (4b).

(4) a. $[_\psi$ exh$_C$ $[_\phi$ you are required to read at least zero books$]]$

b. C = {$\underbrace{\text{you are req. to read more than zero books}}_{\text{excludable}}$, $\underbrace{\text{you are req. to read exactly zero books}}_{\text{excludable}}$}

c. $\psi \Leftrightarrow \Diamond$ you read exactly 0 books $\wedge \Diamond$ you read more than 0 books $\not\Leftrightarrow \top$

Therefore, we conclude that L-analyticity cannot be obviated by embedding under **exh**. Further support for this conclusion comes from the independent observation that the L-analytically true expression ***there is every student** (Barwise & Cooper 1981)) is not salvaged by exhaustification (relative to the alternative **there is a student**), which it would be if exhaustification could obviate L-analyticity. These considerations support the assumption that L-analyticity is indeed a type of ungrammaticality: it behaves like other types of ungrammaticality (e.g. agreement mismatch, case violation), which is also not salvageable by syntactic embedding.

Implications for the semantics of the bare numeral ***zero***: Bylinina & Nouwen (2018) (henecforth *B&N*) argue that the sentence **there are zero students** has two parses, the ungrammatical parse in (5a) and the grammatical parse in (5b), where $\mathbf{exh}_C$ is an exhaustifying operator like that assumed above.

(5) a. *$[_\phi$ there are zero students$]$
b. $[_\psi$ exh$_C$ $[_\phi$ there are zero students$]]$

B&N derive the ungrammaticality of (5a) from the the truth condition in (6a), in conjunction with the assumption that the extension of every plural predicate includes the object $\bot$ (the greatest lower bound $a \sqcap b$ of any two distinct members a and b of a singular predicate) and that the numerosity of this object is 0 (i.e., $\#\bot = 0$). That is, they argue that (5a) is ungrammatical because of its trivial truth condition (6a).

(6) a. $[\![\phi]\!] = 1 \Leftrightarrow \exists x(x \in [\![\textbf{students}]\!] \wedge \#x = 0) \Leftrightarrow \top$
b. $[\![\psi]\!] = 1 \Leftrightarrow \exists x(x \in [\![\textbf{students}]\!] \wedge \#x = 0) \wedge \neg\exists x(x \in [\![\textbf{students}]\!] \wedge \#x > 0) \not\Leftrightarrow \top$

Furthermore, B&N argue that the structure in (5b) is not ungrammatical because it has the non-trivial truth condition in (6b), which they derive from the assumption that C contains the (logically stronger) alternatives $\exists x(x \in [\![\textbf{students}]\!] \wedge \#x = n)$ (for $n > 0$). However, the data discussed in the previous section suggest that L-analyticity cannot be obviated by exhaustification. Therefore, we reject B&N's assumption that the maximality/exhaustive aspect of the meaning of the bare numeral **zero** is syntactically represented.

We close by noting that the nature of the scale that the semantic evaluation of **at least zero** is based on matters for the (un)grammaticality of expressions in which **at least zero** occurs. That is, if **zero** does not refer to the 0 endpoint of the scale, as is the case for the Celsius scale in (7), no deviance arises:

(7) the temperature is at least zero degrees celsius

The contrast between (1b) and (7) mirrors the contrast between ***approximately zero students** and **approximately zero degrees celsius** (Solt 2014). We leave for future research to determine if the triviality of applying an approximator to **zero students** can be conceived of as L-analyticity.

References

Barwise, J., and R. Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4:159–219.

Bylinina, L. and R. Nouwen. 2018. On "zero" and semantic plurality. *Glossa* 3(1):98. 1–23.

Fox, D. 2007. Free choice disjunction and the theory of scalar implicatures. In *Presupposition and Implicature in Compositional Semantics*, ed. U. Sauerland and P. Stateva, 71–120. Palgrave-Macmillan.

Fox, D. and M. Hackl. 2006. The universal density of measurement. *Linguistics and Philosophy* 29. 537–586.

Gajewski, J. 2003. L-analyticity in natural language. Manuscript, MIT.

Solt, S. 2014. An alternative theory of imprecision. In *Proceedings of SALT 24*, 514–533.