

Zero and L-analyticity

Andreas Haida
The Hebrew University of Jerusalem
andreas.haida@gmail.com

Tue Trinh
Leibniz-Zentrum Allgemeine
Sprachwissenschaft
tuetrinh@alum.mit.edu

1 A puzzle

Bylinina & Nouwen (2018) assume that numerals have the weak, ‘at least’ meaning as basic and the strong, ‘exactly’ meaning as derived by way of exhaustification, and propose that **zero** be considered no different from other numerals in this respect.¹

- (1) a. zero students smoked \Leftrightarrow
| $\llbracket \text{students} \rrbracket \cap \llbracket \text{smoked} \rrbracket$ | ≥ 0
b. exh [zero students smoked] \Leftrightarrow
| $\llbracket \text{students} \rrbracket \cap \llbracket \text{smoked} \rrbracket$ | $\geq 0 \wedge$ | $\llbracket \text{students} \rrbracket \cap \llbracket \text{smoked} \rrbracket$ | $\not\geq 1$

The difference between **zero** and other numerals, the authors claim, is that **zero** invokes obligatory exhaustification, because the non-exhaustified meaning is trivial and hence “semantically defective.”² The deviance of (2a) may be taken as supporting evidence for this claim, since it can be explained as resulting from exhaustification being made vacuous by **at least**.³

¹ Assuming that **zero** alternates with other numerals, i.e. that the set of alternatives in this case is {**one student smoked, two students smoked, ...**}.

² A semantic representation of **zero students smoked** which would more immediately reflect Bylinina and Nouwen’s analysis is (i).

(i) $\exists x(\llbracket \text{zero} \rrbracket(x) = \times \llbracket \text{students} \rrbracket(x) = \times \llbracket \text{smoked} \rrbracket(x) = 1)$

These authors assume that the linguistic ontology, i.e. D_e , contains a “bottom element” \perp such that $\llbracket \text{zero} \rrbracket(\perp) = 1$ and $\times \llbracket \alpha \rrbracket(\perp) = 1$ for each expression α of type $\langle e, t \rangle$. We prefer the equivalent but ontologically less controversial representation in (1a), as the discussion below does not hinge on the existence of \perp . Note, also, that in both representations, **zero** ends up being trivially downward and upward entailing in its NP as well as in its VP argument. This means that both representations are compatible with Bylinina and Nouwen’s explanation for **zero**’s inability to license NPIs.

³ Assuming that **at least** alternates with **exactly** and **more than**, i.e. that the set of alternatives in this case is {**exactly zero students smoked, more than zero students smoked**}

- (2) a. #at least zero students smoked
 b. $\text{exh}(2a) \Leftrightarrow (2a) \Leftrightarrow | \llbracket \text{students} \rrbracket \cap \llbracket \text{smoked} \rrbracket | \geq 0$

Suppose Bylinina and Nouwen are right that **zero** requires non-vacuous exhaustification, the question we would like to raise in this squib is (3).

- (3) Is the requirement for non-vacuous exhaustification invoked by **zero** pragmatic or grammatical?

The answer given by Bylinina and Nouwen themselves is that it is pragmatic. We quote from Bylinina & Nouwen (2018: 10): “Unlike other numerals, **zero** invokes exhaustification obligatorily. This is for purely pragmatic reasons.”

We believe there is a problem with this hypothesis: if the requirement were purely pragmatic, all syntactic forms expressing the weak meaning should be equally deviant, but this is not the case, as evidenced by the acceptability of (4), which is semantically equivalent to (2a).

- (4) zero or more students smoked

The contrast between (2a) and (4) suggests that what makes a trivial **zero**-sentence bad is not un informativeness but L-analyticity: the problem is not that the sentence is trivial, but that it is trivial under all non-uniform substitutions of its non-logical vocabulary (Barwise & Cooper 1981; Fintel 1993; Gajewski 2003; Abrusán 2007; Gajewski 2009). Assuming that (4) results from (5) by way of PF-deletion,

- (5) ~~zero students smoked~~ or more than ~~zero~~ students smoked

the sentence is not L-analytical, since one of the two instances of **students** can be substituted by, say, **cats**, resulting in a contingent statement. There is no such substitution for (2a), however, given that the only non-logical term in that sentence is **students** which appears only once.⁴

(Kennedy 2015; Buccola & Haida 2018). We presuppose the definition of **exh** proposed in Fox (2007).

⁴ The following contrast might raise doubt about the deviance of #**at least zero** being due to L-analyticity, as it is implausible that **Celsius** and **Kelvin** are part of the logical vocabulary.

- (i) a. The temperature is at least zero degrees Celsius
 b. #The temperature is at least zero degrees Kelvin

However, we do not need to assume that **Celsius** and **Kelvin** are logical terms to explain this contrast. What we can say is that the presence of **Celsius** vs. **Kelvin** affects whether **zero** denotes the lower endpoint of the scale or not. Our discussion on **zero** is premised

Note that L-analyticity is a formal concept: a sentence containing an L-analytical constituent is expected to be grammatically deviant, whether or not that sentence, as a whole, is uninformative and thus pragmatically deviant. This is why (6a) is unacceptable, even though exhaustification, due to the presence of the universal modal, would make it contingent.

- (6) a. #John is required to talk to at least zero students
 b. $\text{exh}(6a) \Leftrightarrow \text{John is not required to talk to exactly zero students} \wedge \text{John is not required to talk to more than zero students}$

Further evidence for L-analyticity being a formal requirement is the fact that (7a) is deviant, even though exhaustification would make it contingent, assuming that **every** alternates with **some**.⁵

- (7) a. #there is every student
 b. $\text{exh}(7a) \Leftrightarrow \text{there is no student}$

But if unexhaustified **zero** causes deviance by way of the formal constraint against L-analyticity, we would expect that exhaustification cannot alleviate this deviance. In other words, we would make the wrong prediction that (1a) is as bad as (6a) and (7a).

The puzzle, then, is this: if **zero** requires exhaustification to circumvent deviance, that requirement must be grammatical and not pragmatic, but if that is the case, we should expect the deviance to not be circumvented by exhaustification.

2 Some empirical data

2.1 Existential sentences

To corroborate the intuition that **zero**, unlike other numerals, cannot be modified by the adverb **at least**, we conducted an experiment on Amazon MTurk in which 32 English speakers rated the naturalness of 4 sentences

on the understanding that it does denote such a point. Non-standard readings in which it doesn't, as in (ia), are not relevant.

⁵ The triviality of (7a) and contingency of (7b) is derived in (ia) and (ib), respectively, where E is the set of all entities and thus $P \subseteq E$ for all predicates P (Barwise & Cooper 1981).

- (i) a. $(7a) \Leftrightarrow \forall x(x \in S \rightarrow x \in E)$
 b. $\text{exh}(7a) \Leftrightarrow \forall x(x \in S \rightarrow x \in E) \wedge \neg \exists x(x \in S \wedge x \in E) \Leftrightarrow \text{there is no student}$

comparable to (8a) and (8b) on a 4-point scale (= 128 scores for each type of sentence).

- (8) a. there are at least two students in the classroom
 b. *there are at least zero students in the classroom

Figure 1 shows sentences with **at least two** received the highest score 4 ('natural') by $\geq 50\%$ of all subjects, while sentences with **at least zero** received the two lowest scores 2 and 1 ('weird') by $\geq 50\%$ of all subjects. The difference in the means of the scores (3.4 v 2.0), depicted in Figure 2, is highly significant ($p < 2.2^{-16}$).

Figure 1: Boxplot of at least two & at least zero.

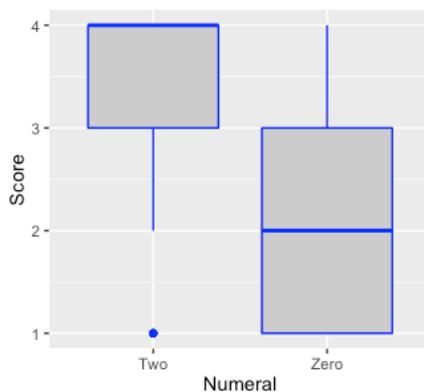
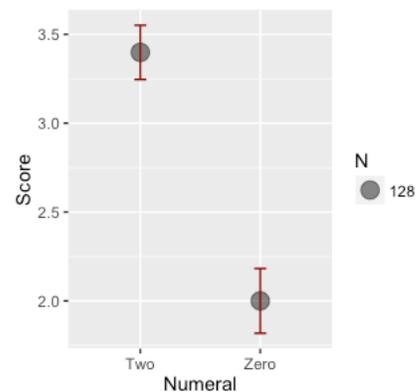


Figure 2: Means of at least two & at least zero.



2.2 Universally quantified sentences

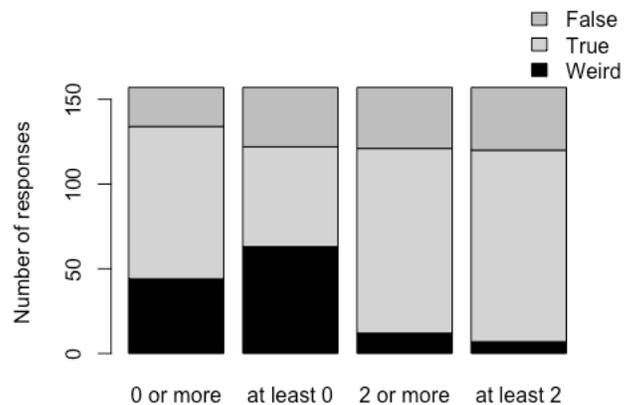
To test the intuition that the relative deviance of #**at least zero** persists under universal quantification, we conducted another experiment on Amazon MTurk. This experiment targeted the contrast between (9a) and (9b), and the contrast between (9a) and the sentences in (10).⁶

⁶ Discriminative judgments for the contrast between (2a) and (4) are hard to come by because of an unavoidable floor effect caused by the uninformative nature of (4). The sentence in (9b) has a distributive inference (i.e., the inference that some humans have zero children and some humans have more than zero children) that makes it informative (cf. the discussion of ex. (6a)). Furthermore, we note that an informal Google search shows a clear difference in the production of the expressions under consideration: a search of, e.g., the phrase **zero**

- (9) a. *every human has at least zero children
 b. every human has zero or more children
- (10) a. every human has at least two relatives
 b. every human has two or more relatives

We asked 157 subjects on Amazon MTurk to judge the sentences in and as either ‘true,’ ‘false,’ or ‘weird’ (1 judgment per subject and sentence). The proportion of ‘weird’ responses to **every...at least zero** is greater than that to its **every...zero or more** counterpart (40% and 28%, respectively, $p = 0.01605$). In contrast, the proportions of ‘weird’ responses to **every...at least two** and **every...two or more** are equal (7% and 12%, respectively, $p = 0.34$) and smaller from **every...at least zero** and **every...zero or more**, see Figure 3.

Figure 3: Number of True-False-Weird judgments.



3 Conclusion

We believe [Bylinina & Nouwen \(2018\)](#) is on the right track but is not the whole story. Questions remain for future research about how the semantics of **zero**, the semantics of **at least**, and the role of exhaustification with respect to L-analyticity are related.

or more times gives 170,000 results, while a search for **at least zero times** only gives 2,780 results.

References

- Abrusán, Martha. 2007. *Contradiction and Grammar: the Case of Weak Islands*: MIT dissertation.
- Barwise, Jon & Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4(2). 159–219.
- Buccola, Brian & Andreas Haida. 2018. Obligatory irrelevance and the computation of ignorance inferences. To appear in *Journal of Semantics*.
- Bylinina, Lisa & Rick Nouwen. 2018. On “zero” and semantic plurality. *Glossa: a journal of general linguistics* 3(1). 1–23.
- Fintel, Kai von. 1993. Exeptive constructions. *Natural Language Semantics* 1(2). 123–148.
- Fox, Danny. 2007. Free choice disjunction and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 71–120. Palgrave-Macmillan.
- Gajewski, Jon. 2003. L-analyticity in natural language. Unpublished manuscript.
- Gajewski, Jon. 2009. L-triviality and grammar. Unpublished manuscript.
- Kennedy, Christopher. 2015. A “de-Fregean” semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8(10). 1–44.