Splitting atoms in natural language*

Andreas Haida
Hebrew University of Jerusalem
andreas.haida@gmail.com

Tue Trinh
UW-Milwaukee
tuetrinh@alum.mit.edu

Abstract

The classic Fregean analysis of numerical statements runs into problems with sentences containing non-integers such as **John read 2.5 novels**, since it takes such statements to specify the cardinality of a set which by definition must be a natural number. We propose a semantics for numeral phrases which allows us to count mereological subparts of objects in such a way as to predict several robust linguistic intuitions about these sentences. We also identify a number of open questions which the proposal fails to address and hence must be left to future research.

Keywords: Numerals, Measurement, Density, Scales, Implicatures

1 A new semantics for numeral phrases

1.1. Standard analyses of numerical statements have roots in Frege (1884) and take these to be, essentially, predications of second order properties to concepts, that is specifications of cardinalities. Thus, the sentence

(1) John read 3 novels

is considered to be a claim about the set of novels that John read, namely that it has three members. The truth condition of (1) is taken to be either (2a) or (2b),

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depending on whether the ‘exact’ or the ‘at least’ meaning is assumed to be basic for numerals.[1]

(2) a. $|\{x \mid x \text{ is a novel} \wedge \text{John read } x\}| = 3$
 b. $|\{x \mid x \text{ is a novel} \wedge \text{John read } x\}| \geq 3$

Let us now consider (3), which we take to be an expression that is accepted as a well-formed sentence of English.

(3) John read 2.5 novels

Extending the traditional analysis of numerical statements to this sentence yields absurdity: (4a) is a contradiction, and (4b) is logically equivalent to (2b).

(4) a. $|\{x \mid x \text{ is a novel that John read}\}| = 2.5$
 b. $|\{x \mid x \text{ is a novel that John read}\}| \geq 2.5$

It is obvious that (3) is neither contradictory nor equivalent to (1). Suppose, for example, that John read *Brothers Karamazov*, *Crime and Punishment*, one-half of *Demons*, and nothing else.[2] In this context, (3) is true and (1) false. The fact that (3) can be true shows that it is not contradictory, and the fact that it can be true while (1) is false shows that the two sentences are not equivalent.[3]

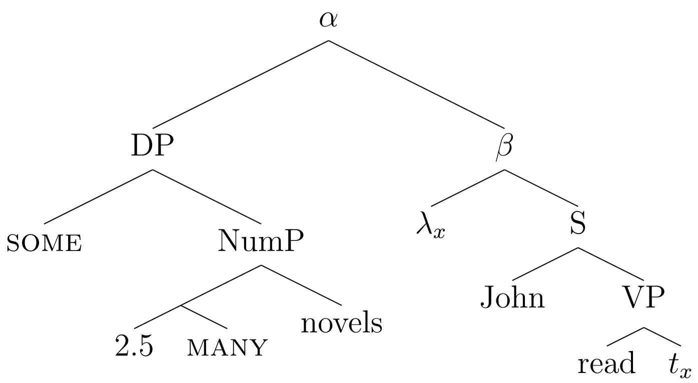
1.2. This squib proposes an analysis of numeral phrases which can account for intuitions about such sentences as (3). First, we will assume the logical form of **John read 2.5 novels** to be (5), where SOME and MANY are covert (cf. Hackl, 2000).

[1] For arguments that numerals have the “at least” meaning as basic see Horn (1972); von Fintel and Heim (1997); von Fintel and Fox (2002); Fox (2007), among others. For arguments that numerals have the ‘exact’ meaning as basic see Geurts (2006); Breheny (2008), among others. Note that the choice between these two views does not affect what we say in this paper, as will be clear presently.

[2] John is a Dostoyevsky enthusiast.

[3] For Frege, the concept of a “concept” entails, as a matter of logic, that it has sharp boundary: “[...] so wird ein unscharf definirter Begriff mit Unrecht Begriff genannt [...] Ein beliebiger Gegenstand Δ fällt entweder unter den Begriff Φ, oder er fällt nicht unter ihn: tertium non datur” (Frege, 1893, §56). Thus, there is no sense in which we can “extend” Frege’s theory to include non-integers: the number of objects which fall under a concept must be a whole number. In fact, Frege considers the reals to be of a different metaphysical category from the naturals, and even made the distinction notationally explicit, writing “2” for the real number two and “~~2~~” for the natural number two (Snyder, 2016; Snyder and Shapiro, 2016).

(5)



Our proposal will consist in formulating a semantics for MANY, leaving other elements in (5) with their standard meaning.[4] This semantics presupposes the fairly standard view of the domain of individuals, $\mathcal{D}_e$, as a set partially ordered by the part-of relation $\sqsubseteq$ to which we add $\emptyset$ as the least element (cf. Link, 1983; Landman, 1989; Schwarzschild, 1996; Bylinina and Nouwen, 2017). The join operation $\sqcup$ and the meet operation $\sqcap$ on $\langle \mathcal{D}_e \cup \{\emptyset\}, \sqsubseteq \rangle$ are defined in the usual way.[5]

We assume that plural nouns denote cumulative predicates, i.e. subsets of $\mathcal{D}_e$ which are closed under $\sqcup$ (cf. Krifka, 1989; Chierchia, 1998; Krifka, 2003; Sauerland et al., 2005; Spector, 2007; Zweig, 2009; Chierchia, 2010). For each predicate A, the set of A atoms, A_{at}, is defined as

(6) $A_{at} := \{x \in A \mid \neg\exists y \,.\, y \sqsubset x \wedge y \in A\}.$

To illustrate, let b and c be the two novels *Brothers Karamazov* and *Crime and Punishment*, respectively. The individual $b \sqcup c$ has proper parts that are novels, hence $b \sqcup c$ will not be in $[\![\textbf{novels}]\!]_{at}$. In contrast, neither b nor c has proper parts that are novels, hence both of these individuals are in $[\![\textbf{novels}]\!]_{at}$. In other words, $[\![\textbf{novels}]\!]_{at}$ contains things that we can point at and say "that is a novel." The semantics we propose for MANY is (7), where d ranges over degrees.

(7) $[\![\text{MANY}]\!](d)(A) = [\lambda x \in \mathcal{D}_e \,.\, \mu_A(x) \geq d]$

We then predict that **John read 2.5 novels** is true iff there exists an individual x such that $\mu_{[\![\textbf{novels}]\!]}(x) \geq 2.5$ and John read x. The term $\mu_{[\![\textbf{novels}]\!]}(x)$ represents "how many novels are in x," so to speak. We want to be able to count novels in

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4 In particular, we assume that the covert SOME has the same meaning as its overt counterpart, which is $[\![\text{SOME}]\!] = [\![\textbf{some}]\!] = [\lambda P \in \mathcal{D}_{\langle e,t\rangle} \,.\, [\lambda Q \in \mathcal{D}_{\langle e,t\rangle} \,.\, \exists x \,.\, P(x) = Q(x) = 1]]$.

5 Here are the definitions, where ι represents, following standard practice, the function mapping a singleton set to its unique element.

(i) a. $x \sqcup y := \iota\{z \mid x \sqsubseteq z \wedge y \sqsubseteq z \wedge \forall z'(x \sqsubseteq z' \wedge y \sqsubseteq z' \rightarrow z \sqsubseteq z')\}$
 b. $x \sqcap y := \iota\{z \mid z \sqsubseteq x \wedge z \sqsubseteq y \wedge \forall z'(z' \sqsubseteq x \wedge z' \sqsubseteq y \rightarrow z' \sqsubseteq z)\}$

such a way that proper subparts of novels, which are not novels, also contribute to the count. To this end, we propose to explicate the measure function μ_A as follows.

(8) $$\mu_A(x) = \begin{cases} \mu_A(y) + 1 & \text{if } a \sqsubset x \text{ and } y \sqcup a = x \text{ for some } a \in A_{at} \\ \mu_a(x) & \text{if } x \sqsubseteq a \text{ for some } a \in A_{at} \\ \# & \text{otherwise} \end{cases}$$

Thus, each A atom which is a subpart of x will add 1 to $\mu_A(x)$. If x is an A atom or a subpart of an A atom, $\mu_A(x)$ will be $\mu_a(x)$, which represents "how much of the A atom a is in x," so to speak. The measure function μ_a is explicated as follows.

(9) For each $a \in A_{at}$,
  a. μ_a is a surjection from $\{x \in \mathcal{D}_e \mid x \sqsubseteq a\}$ to $(0, 1] \cap \mathbb{Q}$
  b. $\mu_a(x \sqcup y) = \mu_a(x) + \mu_a(y)$ for all $x, y \in \text{dom}(\mu_a)$ such that $x \sqcap y = \emptyset$
  c. $\mu_a(a) = 1$

This definition allows us to use any positive rational numbers smaller or equal to 1 to measure parts of an atom, with 1 being the measure of the whole atom.[6] It also allows us to add the measures of parts of an atom provided these parts do not overlap. Thus, chapters 1 and 2 of a novel together with chapters 2 and 3 of the same novel would not add up to 3 chapters of that novel, for example.

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[6] The need for non-integral counting in natural language has been recognized. Kennedy (2015), for example, says the following about #, the measure function which maps objects to number: "Note that # is not, strictly speaking, a cardinality function, but rather gives a measure of the size of the (plural) individual argument of the noun in "natural units" based on the sense of the noun [...]. If this object is formed entirely of atoms, then # returns a value that is equivalent to a cardinality. But if this object contains parts of atoms, then # returns an appropriate fractional or decimal measure [...]" (Kennedy, 2015, footnote 1). However, this is all Kennedy says about the matter. In particular, he does not explicate what he means by "appropriate," and is not concerned with the data that we present below.

The notion of "natural units" refered to by Kennedy in the quote above is due to Krifka (1989), who proposes a function, NU, which maps a predicate P and an object x to the number of natural units of P in x. Like Kennedy, Krifka does not consider the data presented in the next section, and neither does he provide a definition of NU which is explicit enough to relate to them. In fact, Krifka stipulates that NU is an "extensive measure function," on the model of such expressions as **litter of**, which means he actually makes the wrong prediction for the data point presented in 2.2. below. Specifically, Krifka will predict that (11b) must be contradictory as (11a) is.

Thus, what we are doing here is essentially improving upon Kennedy and Krifka, with the improvement being explication in the former and explication as well as correction in the latter case.

2 Some predictions of the proposal

This section presents some intuitions about numerical statements which are predicted by our semantics for MANY. The list is not intended to be exhaustive.

2.1. We predict the observation made at the beginning of this paper, namely that (10a) is neither contradictory nor equivalent to (10b).

(10) a. John read 2.5 novels
b. John read 3 novels

This is because $\mu_{[\![\textbf{novels}]\!]}(x) \geq 2.5$ is neither contradictory nor equivalent to $\mu_{[\![\textbf{novels}]\!]}(x) \geq 3$. To see that $\mu_{[\![\textbf{novels}]\!]}(x) \geq 2.5$ is not contradictory, let b, c, and d be, again, the three novels *Brothers Karamazov*, *Crime and Punishment*, and *Demons*, respectively, and let d' be a subpart of *Demons* which measures one-half of this novel, so that $\mu_{[\![\textbf{novels}]\!]}(d') = \mu_d(d') = 0.5$. Then, $\mu_{[\![\textbf{novels}]\!]}(b \sqcup c \sqcup d') = \mu_{[\![\textbf{novels}]\!]}(c \sqcup d') + 1 = \mu_{[\![\textbf{novels}]\!]}(d') + 1 + 1 = \mu_d(d') + 1 + 1 = 0.5 + 1 + 1 = 2.5$. The non-equivalence follows from the logical truth that $2.5 < 3$ and the fact that there is an x such that $\mu_{[\![\textbf{novels}]\!]}(x) = 2.5$ (as shown above).

2.2. We predict that (11a) is a contradiction but (11b) is not.

(11) a. #John read 1 novel yesterday, and 1 novel today, but he did not read 2 novels in the last two days
b. John read 0.5 novels yesterday, and 0.25 novels today, but he did not read 0.75 novels in the last two days

The first conjunct of (11a)[7] requires two different novels, say b and c, to have been read by John. As $\mu_{[\![\textbf{novels}]\!]}(b \sqcup c) = 2$, the second conjunct of (11a) contradicts the first. On the other hand, suppose John read a subpart of b, call it b', yesterday and read a subpart of c, call it c', today, and suppose that b' measures one-half of b and c' measures one-fourth of c, i.e. $\mu_{[\![\textbf{novels}]\!]}(b') = \mu_b(b') = 0.5$ and $\mu_{[\![\textbf{novels}]\!]}(c') = \mu_c(c') = 0.25$. Then the first conjunct of (11b) is true. However, b' and c', put together, do not make up something which has a subpart that is a novel, or something which is a subpart of a novel. In other words, there is no $a \in [\![\textbf{novels}]\!]_{at}$ such that $a \sqsubset b' \sqcup c'$ or $b' \sqcup c' \sqsubseteq a$, which means $\mu_{[\![\textbf{novels}]\!]}(b' \sqcup c') = \#$, which means $\mu_{[\![\textbf{novels}]\!]}(b' \sqcup c') \not\geq 0.75$, which means the second conjunct of (11b) is true.[8]

[7] Here and below, we refer to the conjuncts of **but**.

[8] Note that our prediction in this case differs from that of Liebesman (2016), who would predict that **John read 0.75 novels** is true in the described context, since Liebesman's proposal, according to our understanding, would allow subparts of different novels to be added, as long as the sum is smaller than 1.

2.3. We predict that (12) is a tautology.

(12) If John read 0.75 novels, and Mary read the rest of the same novel that John was reading, then Mary read 0.25 novels

Suppose John read a portion of b, call it b', which measures three-fourth of b, so that $\mu_{[\![\textbf{novels}]\!]}(b') = \mu_b(b') = 0.75$. Suppose, furthermore, that Mary read the rest of b, call it b'', which is all of that part of b which John did not read. Then the antecedent is true. Now by hypothesis, $b' \sqcup b'' = b$, and $b \in [\![\textbf{novels}]\!]_{at}$. This means $\mu_b(b' \sqcup b'') = \mu_b(b) = 1$. Since b' and b'' do not overlap, i.e. $b' \sqcap b'' = \emptyset$, we have $\mu_b(b' \sqcup b'') = \mu_b(b') + \mu_b(b'') = 1$. And because $\mu_b(b') = 0.75$, we have $\mu_b(b'') = 1 - 0.75 = 0.25$, hence $\mu_{[\![\textbf{novels}]\!]}(b'') = 0.25$, which means the consequent is true.

2.4. We predict that (13) is not a contradiction.

(13) John read 0.5 novels, and Mary read 0.25 of the same novel that John was reading, but John and Mary together did not read 0.75 novels

Suppose John read b' which measures 0.5 of b, and Mary read b'' which measures 0.25 of b. Thus, $\mu_b(b') = 0.5$ and $\mu_b(b'') = 0.25$. The first conjunct is then true. Now let b' and b'' overlap, so that $b' \sqcap b'' \neq \emptyset$. Furthermore, let o be $b' \sqcap b''$ and d' and d'' the non-overlapping parts of b' and b'', respectively. Thus, $b' = d' \sqcup o$, $b'' = d'' \sqcup o$, and $b' \sqcup b'' = d' \sqcup d'' \sqcup o$. This means $\mu_b(b' \sqcup b'') = \mu_b(d' \sqcup d'' \sqcup o) = \mu_b(d') + \mu_b(d'') + \mu_b(o) < \mu_b(d') + \mu_b(o) + \mu_b(d'') + \mu_b(o) = \mu_b(d' \sqcup o) + \mu_b(d'' \sqcup o) = \mu_b(b') + \mu_b(b'') = 0.5 + 0.25 = 0.75$, which means $\mu_b(b' \sqcup b'') < 0.75$, which means hence the second conjunct is true.

2.5. We predict that (14a) is coherent, but (14b) is not.[9]

(14) a. John read (exactly) 0.5 novels
b. #John read (exactly) 0.5 quantities of literature

That (14a) is coherent is, by now, obvious. It will be true if John read, say, half of *Anna Karenina*. What makes (14b) incoherent, then, must lie in the semantics of **quantities of literature**, henceforth **qol** for short. According to the semantics we proposed for MANY, (14b) entails the existence of an individual x such that $\mu_{[\![\textbf{qol}]\!]}(x) = 0.5$, which entails the existence of some $a \in [\![\textbf{qol}]\!]_{at}$ such that $x \sqsubseteq a$. Given that any subpart of a quantity of literature is itself a quantity of literature, we have $[\![\textbf{qol}]\!]_{at} = \{x \in [\![\textbf{qol}]\!] \mid \neg\exists y \sqsubset x \wedge y \in [\![\textbf{qol}]\!]\} = \emptyset$. Thus, there is no $a \in [\![\textbf{qol}]\!]_{at}$, which means there is no x such that $\mu_{[\![\textbf{qol}]\!]}(x) = 0.5$, which means (14b) is false. Furthermore, it is analytically false, which is to say false by virtue

[9] Note that the word **quantity** in (14b) is not intended to mean '200 pages,' or '3000 words,' or any contextually specified quantity of literature. The intended meaning of **quantity** here is the lexical and context-independent one.

of the meaning of the word **quantity**. This, we hypothesize, is the reason for its being perceived as deviant. We will come back to this point in the last section.

2.6. We predict (15), which we claim to be a fact about natural language.

(15) There is no numerical gap in the scale which underlies measurement in natural language

What (15) is intended to say, illustrated by a concrete example, is that to the extent **John read 2.5 novels** is meaningful, **John read 2.55 novels** is too, as well as **John read 2.555 novels**, or any member of $\{$**John read n novels** $\mid [\![\mathbf{n}]\!] \in \mathbb{Q}^+\}$.[10] This follows from the fact that 0.5, as well as 0.55, as well as 0.555, as well as any other rational number in $(0,1] \cap \mathbb{Q}$, are all in the range of μ_a, for any $a \in [\![\mathbf{novels}]\!]_{at}$. This fact, in turn, follows from the fact that μ_a is, by stipulation, a function onto $(0,1] \cap \mathbb{Q}$. Note, importantly, that we cannot guarantee (15) by stipulating, merely, that the set of degrees underlying measurement in natural language is dense. To see that density alone does not exclude gaps, consider the set in (16).

(16) $S := \mathbb{Q}^+ \backslash \{x \in \mathbb{Q} \mid 3 < x \leq 4\}$

This is a dense scale, as between any two elements of S there is an element of S. However, S contains a gap: missing from it are numbers greater than 3 but not greater than 4, for example 3.5. Merely stipulating that the scale is dense, therefore, will not guarantee that **John read 3.5 novels** is meaningful, which we claim is a robust intuition that linguistic theory has to account for.[11]

2.7. On the assumption that overt **many** and the comparative **more** instantiate MANY, we predict that the argument expressed by the sequence in (17) is is invalid.

(17) John read 2.5 novels and Mary read 2 novels. #Therefore, John read more novels than Mary.

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[10] Where $\mathbb{Q}^+$ are the positive rationals. Note that by "meaningful," we mean the sentence has non-trivial truth condition, and licenses inferences, as shown for **John read 2.5 novels** in the last section.

[11] Fox and Hackl (2006), according to our understanding, seems to assume that density of a scale alone guarantees the absense of gaps in it. They claim, for example, that density guarantees that exhaustification of **John has more than 3 children** would negate every element of $\{$**John has more than n children** $\mid \mathbf{n} \in \mathbb{Q} \wedge \mathbf{n} > 3\}$. We quote from page 543 of Fox and Hackl (2006): "Without the UDM [i.e. the assumption that the set of degrees is dense], [...] [t]he set of degrees relevant for evaluation would be, as is standardly assumed, possible cardinalities of children (i.e. 1, 2, 3, ...). The sentence would then assert that John doesn't have more than 4 children [...] If density is assumed, however, [...] the assertion would now not just exclude 4 as a degree exceeded by the number of John's children. It would also exclude any degree between 3 and 4." Taken at face value, this claim is wrong, as is evident from the example in (16).

By the definitions in (7) and (8), the scale $[\lambda x \lambda d.\, [\![\textsc{many}]\!](d)([\![\textbf{novels}]\!])](x)$ is non-monotonic.[12] For instance, if b' is half of *Brothers Karamazov* and c' half of *Crime and Punishment*, then $[\lambda d.\, [\![\textsc{many}]\!](d)([\![\textbf{novels}]\!])](b')(0.5) = 1$ but $[\lambda d.\, [\![\textsc{many}]\!](d)([\![\textbf{novels}]\!])](b' \sqcup c')(0.5) = 0$. Therefore, (17) is not valid, since it would only be valid if the scale were monotonic, i.e. were a scale of comparison (Wellwood et al. (2012)).

This is illustrated in (18). The temparature scale is non-monotonic. Consequently, the temperature scale cannot function as the scale of comparison of the comparative in the second sentence of (18a). Therefore, the sequence of the two sentences in (18a) is an invalid argument. The weight scale, in contrast, is monotonic. Hence, the weight scale can function as the scale of comparison of the comparative in the second sentence of (18b), as evidenced by the validity of the argument expressed by (18b).

(18) a. John ate 90 degree hot spaghetti and Mary 70 degree hot spaghetti. #Therefore, John ate more spaghetti than Mary.
b. John ate 500 grams of spaghetti and Mary ate 200 grams of spaghetti. Therefore, John ate more spaghetti than Mary.

To account for the fact that the arguments in (19) are valid, we tentatively assume that MANY can be restricted to atoms and sums of atoms in equatives and comparatives.

(19) a. John read 3.5 novels and Mary read 2 novels. Therefore, John read more novels than Mary.
b. John read 2.5 novels and Mary read 2 novels. Therefore, John read as many novels as Mary.

This means to say that the scale of comparison of **more than/as many as** in (19) is the scale $[\lambda x \in [\![\textbf{novels}]\!]^{\sqcup}_{at}.\lambda d.\, [\![\textsc{many}]\!](d)([\![\textbf{novels}]\!])](x)$ (where $A^{\sqcup}_{at}$ is the closure of A_{at} under the join operation).

3 Excursus: conditions on predicates

The semantics we propose for MANY, as presented in (7), (8) and (9), requires that for each atom a of a predicate A the measure function μ_a have $(0, 1] \cap \mathbb{Q}$ as its range, and be additive with respect to non-overlapping subparts of atoms.

[12] Let S be a scale, conceived of as a function from entities and degrees to truth values, such that for all x the degree function $S(x)$ is monotonic (i.e. such that $S(x)(d) \to S(x)(d')$ for all d, d' such that $d' \leq d$). Then, the scale S is monotonic iff $S(x)(d) = 1 \to S(x')(d) = 1$ for all d and x, x' such that $x \sqsubseteq x'$ (cf. Krifka 1989; Schwarzschild 2002).

(20) Conditions on μ_a

a. $\text{ran}(\mu_a) = (0, 1] \cap \mathbb{Q}$

b. $\mu_a(x \sqcup y) = \mu_a(x) + \mu_a(y)$ if $x, y \sqsubseteq a$ and $x \sqcap y = \emptyset$

This section details the conditions under which such measure functions μ_a exist, i.e. the conditions on subsets A of $\mathcal{D}_e$ with $A_{at} \neq \emptyset$ such that for each $a \in A_{at}$ there is a function μ_a that satisfies (20a) and (20b). Call such subsets of $\mathcal{D}_e$ "measurable predicates."

Let A be an arbitrary subset of $\mathcal{D}_e$ such that $A_{at} \neq \emptyset$. The first assumption we need to make for A to be a measurable predicate is that all of its atoms are divisible into arbitrarily many discrete parts.[13] This is stated in (21), where $\mathcal{P}_a := \{x \in \mathcal{D}_e \mid x \sqsubseteq a\}$.

(21) For all $a \in A_{at}$ and $n \in \mathbb{N}$, there is a set $S \subseteq \mathcal{P}_a$ such that $|S| = n$, $\bigsqcup S = a$, and $\bigsqcap S' = \emptyset$ for all $S' \subseteq S$ with $|S'| > 1$

It follows from (21) that no A atom a has a smallest part, and also, that there is no smallest difference between two parts of a. This condition is necessary to guarantee that the range of a measure function μ_a can be the rational interval $(0, 1] \cap \mathbb{Q}$, as demanded in (20a).

The second and final assumption we need to make about a measurable predicate A is that its atoms satisfy the condition in (22).

(22) For all $a \in A_{at}$, $\langle \mathcal{P}_a, \sqsubseteq \rangle$ is a σ-algebra on $\langle \mathcal{D}_e \cup \{\emptyset\}, \sqsubseteq \rangle$[14]

σ-algebras are well-known structures of measure theory (see e.g. Cohn 1980) which guarantee, in our case, that the parts of an entity a are measurable in the sense of there being a function μ_a that satisfies (20a) and (20b). In simple words, what we require with (22) is that each $a \in A_{at}$ satisfy the following conditions: (i) the set of parts of a contains a greatest element (trivially satisfied, since a is a part of itself); (ii) for every (proper) part of a, there is another part of a, discrete from the first, such that the two parts together are a; and (iii) countably many parts of a joined together are a part of a. We add another condition to make sure that counting the atoms of a member x of a measurable predicate A is consistent

[13] It seems that a stricter condition might be desirable, viz. that every entity is arbitrarily divisible into discrete parts. However, such a condition would afford a notion of *possible division* of an entity and it is doubtful whether such a notion can be defined independently of the partial order $\langle \mathcal{D}_e \cup \{\emptyset\}, \sqsubseteq \rangle$.

[14] A partial order $\langle A, \sqsubseteq \rangle$ is a **σ-algebra** on a lower bounded partial order $\langle B, \sqsubseteq \rangle$, with $A \subseteq B$, iff (i) it is upper bounded, (ii) closed under complementation, and (iii) closed under countable joins, where $\langle B, \sqsubseteq \rangle$ is **lower bounded** iff $\bigsqcap B \in B$, and $\langle A, \sqsubseteq \rangle$ is **upper bounded** iff $\bigsqcup A \in A$, **closed under complementation** iff for all $x \in A$ there is a $y \in A$ such that $x \sqcup y = \bigsqcup A$ and $x \sqcap y = \bigsqcap B$, and **closed under countable joins** iff for all countable subsets S of A it holds that $\bigsqcup S \in A$.

with measuring all of its subatomic parts. For this to be the case, the atoms of A must be pairwise discrete from each other, as stated in (23).

(23) For all $a, b \in A_{at}$, if $a \sqcap b \neq \emptyset$ then $a = b$

4 Open questions

We end with some open questions for future research. Again, the list below is not intended to be exhaustive.

4.1. Concepts – The semantics we propose for MANY predicts the contrast between (14a) and (14b), repeated in (26a) and (26b) below, because it entails that to be half an A is to be half an A atom. This semantics, as it is, makes the wrong prediction that (24) is false.

(24) The *Unvollendete* is 0.5 symphonies

Let u be the *Unvollendete*. From (8) and (9), it follows that $\mu_{[\![\textbf{symphonies}]\!]}(u) \neq 0.5$, as there is no $a \in [\![\textbf{symphonies}]\!]_{at}$ such that $u \sqsubseteq a$. Obviously, modality is involved: while there is no singular symphony s such that $\mu_s(u) = 0.5$, there could be one, since the last two movements could have been completed. Thus, counting symphonies seems to be about what could be a symphony, not what is actually a symphony. In other words, it is concepts, not predicates, that seem to be at play. This means we should, perhaps, revise our semantics so as to predict that to be half an A is to be half of something which is an A atom in some possible world. There is a possible world, say one where Schubert died at 41 instead of 31, in which the *Unvollendete* is part of a whole symphony, and this is what makes (24) true. However, we do not want to predict, incorrectly, that (25) is true, for example.

(25) *Crime and Punishment* is 0.5 symphonies

Thus, while there certainly is a possible world w in which *Crime and Punishment* is a subpart of a symphony, we want w to be inaccessible from the world of evaluation. Plausibly, specifying the relevant accessibility relation in this particular case amounts to fleshing out the concept of "symphony," and specifying it in the general case, to fleshing out the concept of "concept." We leave this task to future work.

4.2. Analyticities – Suppose John read one quarter of *Brothers Karamazov* and one quarter of *Crime and Punishment*, our semantics of MANY predicts, correctly, that neither (26a) nor (26b) is true.

(26) a. John read 0.5 novels
b. #John read 0.5 quantities of literature

Both sentences claim of something, which does not exist, that John read one-half of it: in the case of (26a), a novel which contains parts of both *Brothers Karamazov* and *Crime and Punishment*, and in the case of (14b), an quantity of literature which contains no subpart that is also an quantity of literature. Our semantics, however, does not predict the contrast in acceptability between (26a) and (26b): while the former is perceived as false, the latter is perceived as deviant. In **2.5**, we said that this contrast has to do with analyticity: it lies in the meaning of the word **quantity** that any subquantity is a quantity, while nothing in the meaning of **novel** rules out a novel which contains parts of both *Brothers Karamazov* and *Crime and Punishment.* Analyticity has been appealed to in explanations of deviance (cf. Barwise and Cooper, 1981; von Fintel, 1993; Krifka, 1995; Abrusán, 2007). However, it has been pointed out that all analyticities are not equal: both (27a) and (27b) are analytically false, but only the latter is deviant.[15]

(27) a. Some bachelor is married
b. #Some student but John smoked

Gajewski (2003) proposes that the kind of analyticity which leads to deviance is "L-analyticity." Thus, while (27a) is analytically false, (27b) is L-analytically false, and therefore is deviant. Discussing Gajewski's notion of L-analyticity will take us beyond the scope of this squib. Hence, we will leave to future research the question whether, and if yes how, sentences such as (26b) can be considered L-analytical.

4.3. Countabilities – Words such as **quantity** have been analyzed as a sort of "classifier" which turns a [−count] noun into a [+count] one (cf. Chierchia, 2010). This analysis is motivated by such contrasts as that in (28).

(28) a. #The vampire drank 2 bloods
b. The vampire drank 2 quantities of blood

Since **blood** is a [−count], it cannot be counted. On the other hand, **quantity of blood** is [+count], therefore it can be. However, such contrasts as that between (28b) and (29), to the best of our knowledge, has not been paid attention to.

(29) #The vampire drank 2.3 quantities of blood

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[15] Assuming that (27a) has the truth condition in (ia) (cf. Heim and Kratzer, 1998) and (27b) the truth condition in (ib) (cf. von Fintel, 1993).

(i) a. $\{x \mid x \in [\![\textbf{bachelor}]\!] \wedge x \in [\![\textbf{married}]\!]\} \neq \emptyset$
b. $\{x \mid x \in [\![\textbf{student}]\!] \wedge x \notin \{\text{John}\} \wedge x \in [\![\textbf{smoked}]\!]\} \neq \emptyset \wedge$
$\forall P(\{x \mid x \in [\![\textbf{student}]\!] \wedge x \notin P \wedge x \in [\![\textbf{smoked}]\!]\} \neq \emptyset \rightarrow \{\text{John}\} \subseteq P)$

The semantics we proposed for MANY, unfortunately, makes no distinction between (28b) and (29): both are predicted to be analytically false. The proposal thus shares with several others the shortcoming of not being able to differentiate between subtypes of [+count] noun phrases. The task remains, therefore, of refining the semantics of MANY as to predict the contrast in question.

It should be noted, in addition, that words like **quantity**[16] may pose a challenge for the theory of measurement proposed in Fox and Hackl (2006). These authors derive the fact that (30a) does not license the scalar implicature (30b)

(30) a. The vampire drank more than 2 quantities of blood
b. ¬The vampire drank more than 3 quantities of blood

from the assumption that the scale mates of **2**, for the deductive system (DS) which computes scalar implicatures, are not the set of natural numbers, but the set of rational numbers. The proposal, therefore, claims that (29) is a scalar alternative of (30a) (see note 11). To the extent that the deviance of (29) is due to this sentence being deemed ill-formed by the DS itself (see Gajewski 2003; Fox and Hackl 2006, and the discussion in the previous subsection), the question arises as to whether DS uses a sentence which it deems ill-formed in its computation. Again, we leave this topic to future work.

4.4. Morphology – The plural vs. singular distinction in number marking languages has usually been considered to mirror the bare vs. classified distinction in classifier languages (cf. Chierchia, 1998; Cheng and Sybesma, 1999). Specifically, plural/bare nouns have been analyzed as denoting "number-neutral" predicates, i.e. sets containing both singularities and pluralities, while singular/classified nouns have been analyzed as denoting "atomic" predicates, i.e. sets containing only singularities. However, with respect to numerical statements involving non-integers in English, a number marking language, and Vietnamese, a classifier language, the correlation falls apart: what is obligatory is a plural noun in English and a classified noun in Vietnamese.

(31) a. John ate 0.5 cake-*(s)
b. John ăn 0.5 *(cái) bánh
John ate 0.5 *(CL) cake

We know of no account for this fact, and leave an investigation of it for future research.

[16] These include **amount** and **fraction**, among possibly others.

(i) a. #The vampire drank 2.3 amounts of blood
b. #The vampire ate 2.3 fractions of the apple

4.5. Reals – We have been assuming that the set of numbers underlying measurement in natural language is $\mathbb{Q}$, the set of rationals. But what prevents us from assuming that it is in fact $\mathbb{R}$, the set of reals? Clearly, that assumption will be true to the extent that sentences containing reals which are not rationals are meaningful. Is (32) meaningful?

(32) John ate π (many) cakes

We have no clear intuition about (32). A confounding factor for such examples as (32) might be that "π" is too "artificial" to be perceived as part of natural language. One might, then, imagine an experiment along the following lines. Let ABC be a cirle on which lie the three points A, B, and C. Let AB be the diameter of ABC. Now suppose a mathematican, say Euclid, uttering the sentence in (33).

(33) If AB is one novel, then ABC is how many novels John read

Obviously, there is no natural language numeral **n** such that Euclid's thought can be expressed as **John read n novels**. The question is whether this thought is, nevertheless, representable by grammar, or more specifically DS, and thus plays a role in inferences such as scalar implicatures (see subsection 4.3). We leave this question to future research.

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