A case for no Ks

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Andreas Haida[1] & Tue Trinh[2]

[1]The Hebrew University of Jerusalem & [2]University of Wisconsin at Milwaukee

Ignorance inference of "at least"

Observation
Sentence (1) licenses the "ignorance inference" that the speaker is ignorant about whether there are exactly two students (Geurts and Nouwen, 2007; Buring, 2008; Schwarz, 2016).

(1) there are at least two students in the classroom
$\rightsquigarrow$ $\neg K(\textit{exactly two}) \wedge \neg K\neg(\textit{exactly two})$
"it's not the case that the speaker believes there are exactly two students in the classroom, and it's not the case that the speaker believes there are more than two students in the classroom"

Two accounts of ignorance

Exhaustification
Every sentence ϕ may be parsed as $exh_C\ \phi$ (cf. Fox, 2007a,b; Chierchia et al., 2012)

(2) Definitions
a. $exh_A\ \phi \Leftrightarrow \phi \wedge \bigwedge\{\neg\psi : \psi \in IE(\phi, A)\}$
b. $\psi \in IE(\phi, A)$ iff $\psi \in \bigcap\{A' \mid A'$ is a maximal subset of A s.t. $\{\phi\} \cup \{\neg\phi' \mid \phi' \in A'\}$ is consistent$\}$

Consequence of Gricean Maxims
The speaker of ϕ is ignorant about relevant propositions which are not settled, i.e. entailed or negated, by ϕ (cf. Grice, 1967).

(3) Assumption about A
If $exh_A\ \phi$ is relevant, every member of A is relevant

Consequence of Gricean Maxims and assumption about A
The speaker of $exh_A\ \phi$ is ignorant about members of A which are not settled by $exh_A\ \phi$ (Kroch, 1972; Fox, 2007a,b; Chierchia et al., 2012).

(4) $exh_A(m \vee s)$ 'John talked to Mary or Sue'
a. $A = \{(m \vee s), m, s, (m \wedge s)\}$
b. $exh_A(m \vee s) \Leftrightarrow ((m \vee s) \wedge \neg(m \wedge s))$
c. $exh_A(m \vee s) \rightsquigarrow \neg Km \wedge \neg K\neg m \wedge \neg Ks \wedge \neg K\neg s$

Horn scale of "at least n"
At least n alternates with **exactly n** and **more than n** (cf. **??**)

Pragmatic account

The syntactic structure of (1) is (5).

(5) $exh_A(\textit{at least two})$
$A = \{\textit{at least two}, \textit{exactly two}, \textit{more than two}\}$

Since $exh_A(\textit{at least two})$ does not settle *exactly two* or *more than two*, it licenses the inference that the speaker is ignorant about *exactly two*.

(6) $exh_A(\textit{at least two})$
$\Leftrightarrow$ *at least two*
$\rightsquigarrow \neg K(\textit{exactly two}) \wedge \neg K\neg(\textit{exactly two})$

Semantic account

The syntactic structure of (1) is (7).

(7) $exh_A(K(\textit{at least two}))$
$A = \{K(\textit{at least two}), K(\textit{exactly two}), K(\textit{more than two})\}$

It follows from the literal meaning of $exh_A(K(\textit{at least two}))$ that the speaker is ignorant about *exactly two* (cf. Meyer, 2014; Buccola and Haida, 2017).

(8) $exh_A(K(\textit{at least two}))$
$\Leftrightarrow K(\textit{at least two}) \wedge \neg K(\textit{exactly two})$
$\wedge\ \neg K\neg(\textit{exactly two})$

Is K syntactically represented?

L-analyticity

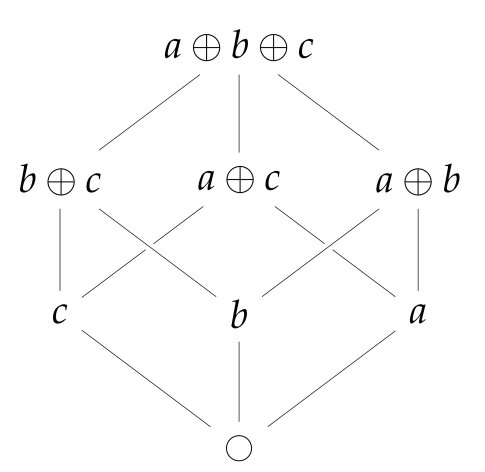
Deviance may result from the sentence being "L-analytical," i.e. tautological or contradictory purely by virtue of the configuration of logical constants contained in it (Barwise and Cooper, 1981; Fintel, 1993; Gajewski, 2003; Chierchia, 2006; Abrusa n, 2007; Gajewski, 2009; Abrusa n, 2011)

(9) a. there is a student $\Leftrightarrow \exists x(x \in S \wedge x \in E)$
b. *there is every student $\Leftrightarrow \forall x(x \in S \rightarrow x \in E) \Leftrightarrow_L \top$

(10) a. everyone but Bill danced
$\Leftrightarrow \forall x(x \notin \{b\} \rightarrow x \in D) \wedge \forall P(\forall x(x \notin P \rightarrow x \in D) \rightarrow \{b\} \subseteq P)$
$\Rightarrow \forall x(x \notin \{b\} \rightarrow x \in D) \wedge \neg\forall x(x \notin \varnothing \rightarrow x \in D)$
b. *someone but Bill danced
$\Leftrightarrow \exists x(x \notin \{b\} \wedge x \in D) \wedge \forall P(\exists x(x \notin P \wedge x \in D) \rightarrow \{b\} \subseteq P)$
$\Rightarrow \exists x(x \notin \{b\} \wedge x \in D) \wedge \neg\exists x(x \notin \varnothing \wedge x \in D)$
$\Leftrightarrow_L \bot$

Zero

We adopt the theory proposed in Bylinina and Nouwen (2017), according to which *every* plural noun has in its the denotation a special element, $\bigcirc$, whose atoms count 0.



$[\![\text{two students}]\!] = [\lambda x.\ x \in [\![\text{student}]\!] \wedge \#x = 2]$
$= \{a \oplus b, b \oplus c, a \oplus c\}$

$[\![\text{zero students}]\!] = [\lambda x.\ x \in [\![\text{student}]\!] \wedge \#x = 0] = \{\bigcirc\}$

(11) there are n students $\Leftrightarrow \exists x(x \in [\![\text{students}]\!] \wedge \#x = n)$

Parsing (12) with exh_A, as in (12b) rescues it from being an L-analytical sentence, assuming **zero** alternates with other numerals.

(12) there are zero students
a. $[_S$ there are zero students$]$
$\Leftrightarrow \exists x(\#x = 0 \wedge x \in [\![\text{students}]\!]) \Leftrightarrow_L \top$
b. $[_S\ exh_A\ [_S$ there are zero students$]]$
$\Leftrightarrow \exists x(\#x = 0 \wedge x \in [\![\text{students}]\!]) \wedge \neg\exists x(\#x > 0 \wedge x \in [\![\text{students}]\!]) \not\Leftrightarrow_L \top$

A novel observation

(13) a. there are at least two students in the classroom
b. *there are at least zero students in the classroom

Figure 1: Boxplot of **at least 2** and **at least 0**

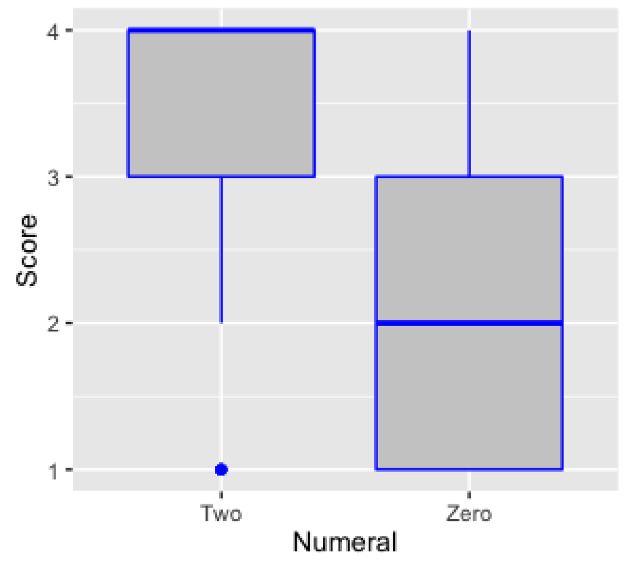
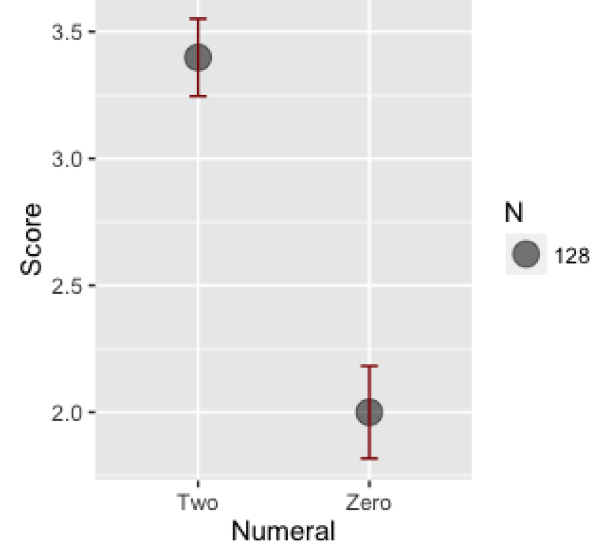


Figure 2: Means of **at least 2** and **at least 0**



K is not syntactic

Suppose K is not syntactically represented. Then the available parses for (13b) are (14a) and (14b).

(14) a. $\textit{at least zero} \Leftrightarrow \top_L$
b. $exh_A(\textit{at least zero}) \Leftrightarrow \top_L$

(15) $exh_A(\textit{at least zero}) \Leftrightarrow \textit{at least zero}$
as $exh_A(\textit{at least zero})$ does not settle *more than zero* and *exactly zero*

K is syntactic

Suppose K is syntactically represented. Then, (16a) and (16b) would be available as parses for (13b).

(16) a. $K(\textit{at least zero}) \Leftrightarrow \top_L$
b. $exh_A(K(\textit{at least zero})) \not\Leftrightarrow \top_L$

(17) $exh_A(K(\textit{at least zero}))$
$\Leftrightarrow K(\textit{at least zero}) \wedge \neg K(\textit{exactly zero})$
$\wedge \neg K(\textit{more than zero})$

Thus, if K is syntactically represented, there would be a parse for (13b) which is not L-analytical, i.e. not deviant. This means that the deviance of (13b) is evidence that

K is not syntactically represented?

A prediction

We predict that the *meaning* of (13b) can be felicitously expressed by a non-L-analytical sentence, such as (18a), whose LF is (18b) (Hurford, 1974; Chierchia et al., 2012; Fox and Spector, 2018).

(18) a. there are zero or more students
b. exh_A(there are zero students) or (there are more than zero students)

A Google search of, e.g., the phrase **0 or more times** gives 170,000 results, while **at least 0 times** only gives 2,780 results.

Are there better theories of 'zero'?

Suppose numerals have a two-sided meaning as a matter of semantic content (Breheny 2008, Geurts 2006, Kennedy 2015). We correctly derive that **there are zero students** is non-tautological, and that **there are at least zero students** is L-tautological.

(19) a. there are 0 students $\Leftrightarrow exh_C$(there are 0 students)
$\Leftrightarrow \max\{n \mid \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]\} = 0 \not\Leftrightarrow \top$
b. there are at least 0 students $\Leftrightarrow exh_C$(there are at least 0 students)
$\Leftrightarrow \max\{n \mid \exists x[x \in [\![\text{students}]\!] \wedge \#x = n]\} \geq 0 \Leftrightarrow_L \top$

However, we still derive, incorrectly, that the deviance of **at least zero** is obviated under universal quantification:

(20) $\text{exh}_C\ (K(\text{there are at least 0 student}))$
$\Leftrightarrow K(\max\{n \mid \exists y[y \in [\![\text{students}]\!] \wedge \#y = n]\} \geq 0])$
$\wedge \neg K(\max\{n \mid \exists y[y \in [\![\text{students}]\!] \wedge \#y = n]\} = 0])$
$\wedge \neg K(\max\{n \mid \exists y[y \in [\![\text{students}]\!] \wedge \#y = n]\} > 0])$
$\not\Leftrightarrow \top$

The logical status of scales

(21) a. #There are 0 students in the classroom
b. The temperature is at least 0 degrees Celsius
c. #The temperature is at least 0 degrees Kelvin

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